Menu Costs, Aggregate Fluctuations, and Large Shocks

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Abstract

We document that the aggregate price level responds flexibly and asymmetrically to large positive and negative value-added tax changes. We present a price-setting model with menu costs, trend inflation and fat-tailed product-level shocks that is consistent with these observations. The model predicts a flexible price-level response to standard monetary policy shocks because it anticipates a large number of firms on the verge of price adjustment and far from their optimal prices when the shock hits.

JEL: E31, E52
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1 Introduction

How effective is monetary policy when price stickiness is caused by menu costs of price adjustments?\(^1\) Access to detailed micro-level pricing data has revitalized research into this fundamental question. New evidence demonstrated that price changes are not only infrequent but also highly dispersed.\(^2\) Quantitative analysis revealed that even if the frequency is low, dispersion remains a key determinant of monetary non-neutrality through its influence on which prices adjust when a money shock hits. Golosov and Lucas (2007), assuming low dispersion, found near neutrality, while

\(^{1}\)Mankiw (1985) and Caplin and Spulber (1987).
\(^{2}\)See e.g. Klenow and Malin (2010).
Midrigan (2011), matching high dispersion, found strong non-neutrality. This literature predominantly relies on observations in environments where aggregate shocks are small. In this paper, we confront these models with observations on price responses to aggregate shocks that are large. These rare observations illuminate key features of the environment, which normally stay hidden.

We use large aggregate shocks to learn about a key source of price dispersion: the underlying distribution of product-level shocks, which gets revealed when the majority of prices change. We show that small changes in this distribution can have a large impact on monetary non-neutrality. In particular, we set up a menu cost model with a novel form of distribution that generalizes the setup of Golosov and Lucas (2007) and Midrigan (2011). The data favor a fat-tailed distribution close to that used in Midrigan (2011), and the model matches key moments of price changes both during normal times and after large shocks. Money, however, is near neutral as in Golosov and Lucas (2007). The reason is the impact of the distribution on the optimal price-change size of products that are on the verge of adjustment.

We document micro-level price responses to three large value-added tax (VAT) changes, which occurred in Hungary in 2004 and 2006. We show that prices responded (i) flexibly and (ii) asymmetrically to subsequent positive and negative tax changes. In particular, a five-percentage-point tax increase raised the monthly frequency of price changes of affected food products to 60 percent from 13 percent, while it increased the frequency to only 27 percent after a five-percentage-point tax decrease. Furthermore, in the months with tax changes, price changes became (iii) smaller: the average size of absolute price changes declined, and (iv) more concentrated, in the sense that the distance between the first and third quartile of the absolute price change distribution narrowed and the kurtosis of price changes increased. These four new observations motivate our quantitative analysis.

Price responses to large VAT changes are well-suited to assess the predictions of menu cost models. First, the tax changes are measurable aggregate cost shocks, with clear predictions on their impact on the price level in standard flexible-price models. Second, menu costs affect this the price-level impact, because in Hungary posted prices include VAT, and stores need to post new prices (i.e., pay the menu cost) if they choose to incorporate the tax change into their prices. Third, qualitative predictions of menu cost models are in line with our first two observations. The price-change frequency increases (observation i), because numerous firms are willing to pay the menu cost in the face of the large tax change. Additionally, the presence of positive trend inflation (Ball and Mankiw, 1994) can explain the asymmetry (observation ii). We ask whether menu cost models

See also Alvarez et al. (2016).

The reason is that after a negative shock, firms can save their menu cost if they keep their nominal price constant and allow the trend inflation (i.e., price increases of their competitors) to reduce their relative price towards its optimum level for free. In contrast, inflation does not support real adjustment after a positive shock, so then more firms need to change their prices.
can match these observations also quantitatively, and what our observations can teach us about relevant characteristics of these models.

We present a quantitative menu-cost model with heterogeneous firms that face product-level (idiosyncratic) shocks. As mentioned before, the focus of our analysis is on the shape of this shock distribution. This shape is important because it can determine the extent of monetary non-neutrality, as previous research has shown. Golosov and Lucas (2007) found near money neutrality assuming a normal (Gaussian) distribution. In contrast, Midrigan (2011) found strong non-neutrality using a fat-tailed Poisson distribution\(^5\). In our analysis, we assign the distribution a flexible parametric form, a mixture of two normal distributions, which conveniently nests the normal and the Poisson distributions but also allows for a continuum of intermediate cases. We use observations in months with and without tax changes to infer the realistic shape of this distribution and analyze its implications on monetary non-neutrality.\(^6\)

Why are the price responses to large tax shocks informative about the realistic shape of the idiosyncratic shock distribution? The large shock pushes a large number of firms over their inaction thresholds, and therefore the normally unobserved price gap distribution temporarily reveals itself. For example, the high kurtosis (observation iv) of the price change distribution in these months immediately points towards an underlying distribution with excess kurtosis. As we show in the paper, our additional observations about the extent of frequency, asymmetry, and size of price changes in the months with tax changes also depend on the shape of the underlying distribution. Our question is whether our model with mixed normal shock distribution generates predictions that match these observations quantitatively.

To answer this, we bring our model to the data. As a first step, we calibrate its key parameters to match the low frequency, large absolute size and wide dispersion of price changes that we observe in normal months without any large tax changes. The model closely matches the cross-sectional distribution of price changes. The shape of the product-level distribution favored by the data is close to the Poisson distribution and similarly features fat tails. As a second step, we simulate pricing responses in our calibrated model to VAT changes. We set the sizes of these tax changes to match those observed in the data. We find that our baseline model predicts both the magnitude and asymmetry of the observed price change frequency remarkably well in the months with tax changes. Furthermore, the model can match the declining size, the increasing kurtosis and declining

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\(^5\)We define Poisson distribution as a normal shock arriving with a specific Poisson rate as in Midrigan (2011); Vavra (2014). See also Gertler and Leahy (2008).

\(^6\)To further improve the realism of our model, we also assume multi-product firms with two products, which they can re-price for a single menu cost. As the price change of the second product is ‘free,’ this assumption generates frequent small price changes, which we observe in the data. Finally, we also introduce positive trend inflation in our model. The trend inflation is something we observe in our sample, and it has the potential to generate asymmetric responses to positive and negative aggregate shocks. We solve the model numerically with standard global solution methods.
interquartile range of the price change distribution, in line with all of our empirical observations. This is remarkable, as our calibration strategy leaves us with no free parameters to target pricing moments in the months with tax changes.

Equipped with a model that matches relevant pricing facts both during normal and tax-changing months, we set out to measure the magnitude of monetary non-neutrality predicted by the model. We find that the model generates small and temporary real effects to simulated monetary policy shocks as in Golosov and Lucas (2007). The reason is that our preferred distribution generates high Caplin-and-Spulber-type (1987) selection of price changes. In other words, there are a large number of firms on the verge of price adjustment and far from their optimal price when the aggregate shock hits. Therefore, the aggregate shock interacts with the cross-sectional price-gap distribution and induces a flexible price level response with weak real effects. The near-neutrality of money in our model might appear puzzling given that our calibrated mixed normal distribution is close to the Poisson distribution, which predicts large and persistent real effects. As we show, however, the selection effect is determined by the non-linear interaction of the magnitude of the menu costs and the number of firms on the verge of price adjustment. Both of these variables increase as we move from the Poisson distribution towards our preferred distribution, and their non-linear interaction means that a small deviation is sufficient for a sizable decline in the level of predicted non-neutrality. Therefore, the aggregate predictions of these models are highly sensitive to the exact shape of the idiosyncratic shock distribution. Capturing conventional moments, like the frequency, the size, or the kurtosis of the price-changes might be insufficient to identify the shape; capturing additional dispersion measures, like the inter-quartile range in normal times, or frequency measures conditional on large shocks might be equally important. Our model predicts high selection because of the novel shape of the idiosyncratic shock distribution, which helps it to capture the shape of the cross-section of price distribution and predict pricing responses to large shocks well. We conclude that high selection and near monetary neutrality is a robust prediction of menu-cost models across a wide range of realistic idiosyncratic shock distributions.

Related literature Price rigidities are key frictions in standard New-Keynesian macroeconomic models (Christiano et al., 2005; Smets and Wouters, 2007). The most widely applied price-setting model (Calvo, 1983) assumes that prices can only be adjusted at randomly arriving occasions. The assumption is analytically convenient as it facilitates aggregation, but the random assignment of price changes rules out selection and implicitly increases monetary non-neutrality predicted by the framework. Menu costs of price adjustment is another frequently cited micro-foundation of price rigidities, which is consistent with relevant stylized facts documented in micro-level data (see, for example, Bils and Klenow, 2004; Klenow and Malin, 2010). As a consequence, it is crucial to understand whether the Calvo (1983) model’s aggregate predictions are in line with those of menu cost models. We contribute to this literature by presenting a menu cost model that matches stylized
facts both in normal periods and after large aggregate shocks and show that the model implies near monetary neutrality contrary to the Calvo (1983) model.

Previous research has documented price-setting responses to VAT changes (see, for example Gagnon et al., 2012; Alvarez et al., 2006). These studies have also found significant increases in the frequency of price changes in the months with tax changes, inconsistently with classic time-dependent pricing models. We add to the existing VAT evidence by documenting asymmetric responses to positive versus negative VAT changes,\footnote{VAT changes are arguably more easily measurable, exogenous, identifiable shocks than money-growth shocks, which have also been used to document asymmetric aggregate inflation responses to monetary shocks in previous studies (see, for example, Cover, 1992; Ravn and Sola, 2004). VAT shocks also have the advantage of being an aggregate shock, hitting multiple products simultaneously. Asymmetric reactions to product-level shocks – documented, for example, by Peltzman (2000) – are not as informative because they could also be in line with standard time-dependent models (like those in Calvo, 1983) with trend inflation. This is because trend inflation induces firms to front-load their price adjustment by setting it above their static optimum. In doing so, they can keep their gradually decreasing real price close to the optimum throughout their price spell. As a result, they respond to a positive idiosyncratic shock with a larger price increase and to a negative shock with a lower price decrease (in absolute terms). This firm-level asymmetry, however, does not necessarily translate into aggregate inflation asymmetry. In time-dependent pricing models with an exogenous probability of price change, for example, the firm-level asymmetry cumulates precisely to the trend inflation, while the additional inflation effects of the aggregate shocks remain symmetric.}, and features of the cross-section of price-changes in months with tax changes. These observations can help the calibration of price-setting models. We show that a menu cost model with a suitably chosen mixed normal idiosyncratic shock distribution and trend inflation can match these observations.

In menu cost models the shape of the idiosyncratic shock distribution plays a crucial role in determining the extent of monetary non-neutrality (Midrigan, 2011; Elsby and Michaels, 2014). It does this through influencing the selection effect, i.e., the distribution of price gaps on the verge of adjustment. Alvarez et al. (2016) finds that the kurtosis of the price-change distribution is a sufficient statistic to measure this effect in a wide range of menu cost models with normally distributed idiosyncratic shocks. Their conclusion, however, does not extend to cases without the normality assumption, like in Midrigan (2011), Gertler and Leahy (2008), or in our paper with a mixed-normal distribution. Intuitively, an unidentified idiosyncratic shock distribution gives too much degree of freedom to influence selection, which depends only on a narrow range of the price change distribution around the inaction thresholds, without affecting the kurtosis of the whole price-change distribution (for an instructive example, see Caballero and Engel, 2007). Our distribution is a realistic example that helps the model to match the price-change distribution including its high kurtosis both during normal times and after large aggregate shocks and still leads to near money neutrality.\footnote{Alvarez et al. (2016) confirm our results in a continuous time framework (see their Section V.B).}

**Structure:** In Section 2, we present our data and the evidence on price setting after large VAT shocks. In Section 2.3, we present our model, and in Section 3, we calibrate its parameters. In Section 4, we show that the model effectively predicts pricing responses to large aggregate shocks.
In Section 5, we derive the extent of monetary non-neutrality in the model, analyze its causes in a simplified model and demonstrate its robustness. We conclude in Section 6.

2 Empirical evidence

In this section, we describe our micro-price data and discuss the details of the value-added tax changes that form the empirical basis of our analysis.

2.1 Tax changes in Hungary

The Hungarian government implemented three major value-added tax (VAT) changes between 2004 and 2006. First, to conform to EU regulations on minimum VAT rates, it increased the preferential rate from 12 to 15 percent in January 2004. Second, to simplify the tax system, it sequentially closed the gap between two different VAT rates in 2006. In January, before the general elections, it decreased the standard rate from 25 to 20 percent. Then in September, after the election, it increased the preferential rate from 15 to 20 percent. Importantly, the events were driven by factors unrelated to price developments, so they can be treated as exogenous variation in firms' costs. Furthermore, all of these changes were, for all practical purposes, permanent.

The events offer suitable opportunities to assess the performance of menu cost models. First, changes in value-added taxes are easily measurable and transparent nominal cost shocks, which can be easily simulated in standard pricing models. Second, in Hungary gross prices are posted, so firms need to actively reprice their products to incorporate value-added tax changes - differently from standard US practice where prices are posted net of sales taxes. Third, tax changes were large: they reached 3.5-6 times the standard deviation of inflation in regular months and caused salient changes in the price-setting behavior. Fourth, the tax changes were pre-announced, and widely publicized, which minimized the information frictions about their size and timing. Importantly, during the pre-crisis period, the country enjoyed a stable macroeconomic environment with stable growth, low inflation rates, and stable exchange rates. Furthermore, monetary policy refrained from responding to the tax changes: The inflation-targeting central bank had expressed in advance that it was “seeing through” the direct effects of the tax changes because they affected only the price level with temporary impact on measured inflation.

We restrict our attention to the processed food sector, for two main reasons. First, in this sector, the composition of the subgroups facing the two distinct value-added tax rates are very similar.  

\[^9\] An example of similar products facing different tax rates is 'cookies' facing the lower and 'chocolate-chip cookies' facing the higher rate. The two groups have similar price-setting moments: the average yearly inflation rates were 4.3 percent versus 4.1 percent; the steady-state frequencies of monthly price changes were 11.5 percent versus 12.9 percent; the average absolute sizes of price changes were 9.7 percent versus 10.8 percent; the kurtoses were 3.98 versus 3.96; and the interquartile ranges of price changes are 8.1 percent and 8.3 percent, respectively.
This permits us to disregard composition effects and contributes to a clean comparison of the price-level effects of the positive and negative tax changes, which is a focus of our paper.\textsuperscript{10} Second, this is the largest homogeneous sector in our sample, with a CPI-weight of 16.1 percent. Notably, the moments of the price-change distribution in this sector are close to the US retail store data used, for example, by Midrigan (2011).

To motivate our prospective analysis, Figure 1 shows the annualized monthly inflation rates of the affected processed food products around the positive and negative tax changes in 2006 (the latter on a reverse scale for the ease of comparison). The figure highlights that the price level responded flexibly and asymmetrically to the tax changes. While the price-level increase approximately equals the annualized cost increase, the price-level decrease is only around one-third of the tax decrease.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Monthly inflation (processed food sector, Hungary)}
\end{figure}

The figure plots the annualized monthly inflation rates in the processed food sector around the positive (in September 2006) and negative (in January 2006, on a reverse scale) 5 percentage-point value added tax changes.

\textsuperscript{10}We provide further evidence on the lack of composition bias in Appendix C.2, by studying the inflation effects of yet another five-percentage-point VAT increase in July 2009, which affected predominantly the products that experienced a VAT-decrease earlier. We do not directly include this VAT change in our baseline analysis, because of the severe cyclical downturn during its implementation.
2.2 The data

In this section, we show that in our Hungarian dataset price changes are similarly infrequent, large and dispersed during normal months as those analyzed in analogous data in other countries (Klenow and Malin, 2010).

We use a Hungarian dataset of micro-level product prices underpinning the construction of the consumer price index. Unless indicated otherwise, we concentrate on the pre-crisis sample between January 2002 and December 2006. In the analyzed processed food sector, we observe the prices of 128 different 5-digit products \((i)\),\(^{11}\) and each product is observed in 123 stores \((s)\) on average, each month.\(^{12}\) We calculate price changes for each product-store combinations for each month: 

\[
\Delta p_{ist} = \log P_{ist} - \log P_{ist-1}
\]

We sales-filter our data to focus on regular price changes. There is an ongoing debate in the literature about the importance of sales in determining the flexibility of the price level (see, for example, Kehoe and Midrigan, 2015; Kryvtsov and Vincent, 2017). By sales-filtering the data we take a conservative approach. Our model implies near-neutrality of money even without sales; the extent of neutrality would be even higher if sales provided an additional adjustment margin for firms and would make the price level even more flexible. To calculate regular price changes, we first exclude price changes that are flagged as sales by the price collectors in the data. Second, we drop any remaining price changes that are (i) at least 10 percent, and (ii) are completely reversed within a single month.\(^{13}\)

To calculate moments of a representative product, we first calculate each moment across all stores \(s\) for each 5-digit product \(M_{it}\), and then aggregate them using the 2006 expenditure-based CPI-weights \((M_t = \omega_i M_{it})\). This way, we avoid pooling potentially heterogeneous product-level price-change distributions.\(^{14}\) To calculate steady-state moments, we seasonally adjust the time-series using month dummies and filter out the immediate impact of value-added tax changes by subtracting the coefficients of time-dummies that take value 1 in the months with tax changes.

\(^{11}\)These categories are closest to the entry-level items in the US CPI. From the products, 98 were affected by the tax increase and 30 by the tax decrease.

\(^{12}\)The number of item replacements and substitutions are small.

\(^{13}\)The CPI microdata we use record actual posted prices (not unit value indices as most barcode datasets), so they are less susceptible to the kind of measurement error analyzed in Eichenbaum et al. (2014). The processed food products we use are not considered ‘problematic’ either by Eichenbaum et al. (2014). The only processed food they consider problematic (cigarettes) are excluded from our sample because they were subject to frequent large excise tax changes. We mitigate the impact of potential measurement error further by censoring price change observations at 100 percent in absolute value. Alvarez et al. (2016) provide evidence that such censoring of CPI data brings measured moments closer to those observed in online data, which is presumably less susceptible to measurement error.

\(^{14}\)An alternative would be to calculate moments of a pooled distribution of price changes that are standardized at the product-store level (see Klenow and Kryvtsov, 2008; Midrigan, 2011; Alvarez et al., 2016). As standardization at the product level is naturally achieved by our method, we do not expect this choice to influence our results.
2.3 Moments of price changes

In this section, we present key moments of regular price changes in normal months and during the months of tax changes that motivate our analysis. The first column of Table 1 lists moments during regular months (steady state).

Table 1: Moments of regular price-change distribution

<table>
<thead>
<tr>
<th></th>
<th>Steady State</th>
<th>Tax changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Midr. (2011)</td>
</tr>
<tr>
<td>Frequency</td>
<td>12.6%</td>
<td>11.6%</td>
</tr>
<tr>
<td>Absolute Size</td>
<td>9.9%</td>
<td>11%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.98</td>
<td>4.02</td>
</tr>
<tr>
<td>Abs. interquartile range</td>
<td>8.13%</td>
<td>8%</td>
</tr>
<tr>
<td>Inflation</td>
<td>4.23%</td>
<td>0%</td>
</tr>
</tbody>
</table>

The table shows key moments of the price change distribution during normal months and in the months with tax changes. In particular, it shows the frequency of price changes, the kurtosis of the regular price-change distribution, the average size and interquartile range of the absolute price change distribution and the annualized monthly inflation rate.

The frequency measures the non-zero regular price changes as a fraction of observed prices. Regular prices infrequently change, only around once every eight months. When they do, the price changes are sizeable; they are close to 10 percent. This can be seen in our size measure, which calculates the average absolute size of all non-zero price changes. Furthermore, we can observe a sizable dispersion of regular price changes: many price changes are small (10 percent of the price changes are less than 3 percent in absolute value, not shown), but some price changes are large (10 percent is larger than 19 percent). Furthermore, the difference between the first and the third quartile of the absolute regular price-change size distribution is more than 8 percent. The kurtosis of the regular price-change distribution is around 4, which is significantly larger than the kurtosis of the normal distribution (3). The second column in Table 1 confirms that our data are standard: the moments of our regular price-change distribution mimic those observed in a major US grocery-store chain Dominick’s analyzed by Midrigan (2011). The inflation rate is around 4.2 percent in Hungary in the processed food sector during our sample period. These observations motivate the key features of our quantitative model and form the basis of our calibration.

The last three columns of Table 1 shows the evolution of the key pricing moments in the months with large VAT changes. A couple of observations stand out. First, the frequency of price changes among affected products increases substantially in these months. After the VAT increases, they

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15 Alvarez, Le Bihan, Lippi (2016) find that a kurtosis of 4 is a reasonable estimate in the US for a general set of products.
increase from 12.6 percent to 52 and 62 percent after the 3 and 5 percentage-point increases, respectively. This observation indicates that prices are not fully flexible, because a sizable fraction of affected prices stays unchanged (48 and 38 percent) despite the large cost change. At the same time, these observations would also be in contrast with time-dependent pricing models that assume a fixed frequency of price changes. Second, in line with the asymmetry of the inflation response highlighted in Figure 1, the frequency increases only to 27 percent after a 5 percentage point VAT decrease. Such asymmetry is a classic prediction of menu cost models with trend inflation (Ball and Mankiw, 1994). After a negative shock, firms can save their menu cost by keeping their nominal price constant and allowing the trend inflation to reduce their relative price to its optimum level for free. Inflation, however, does not help them after a positive shock. Even though models with menu costs have a good chance of capturing both the flexibility and the asymmetry of the price level responses after large shocks, we show later that standard models are unable to match the quantitative magnitude of these observed responses. Third, the table shows a drop in the average size of price changes, increases in kurtosis and sizable declines in the interquartile range of the absolute price changes in the months of VAT changes. These observations are informative about the underlying cross-sectional distribution of desired price changes or price gaps. We argue below that these observations are in line with a price-gap distribution with excess kurtosis. We aim to develop a menu-cost model that comes close to capturing these key micro-level pricing facts quantitatively. We turn to set up such a model below.

The model

Motivated by the observations presented in the previous section, we build a non-linear dynamic stochastic general-equilibrium multiproduct menu-cost model in which firms face idiosyncratic shocks. The focus of our analysis is the shape of idiosyncratic shock distribution. We introduce a flexible parametric form to model the shape and ask how it influences the extent of monetary non-neutrality in the model.

The model consists of a representative consumer, a continuum of firms, and a government. The consumer supplies labor, consumes a bundle of differentiated goods and saves. Firms use labor to produce multiple differentiated products and face idiosyncratic shocks. Firms need to pay a fixed menu cost to change the price of at least one of their products. Firms are assumed to satisfy all demand at their posted price. The government sets an exogenous process for the money supply, and the level of value-added tax rates.\footnote{In our framework with no intermediate production, the value-added tax is equivalent to sales tax.}
2.3.1 Consumers

The representative consumer consumes a Dixit-Stiglitz aggregate ($C_t$) of a basket of multiple goods $g$ ($g = 1, 2, \ldots, G$) purchased from firms $i$ ($i \in [0, 1]$), and supplies labor $L_t$ to maximize the expected present value of her utility:

$$\max_{C_t(i,g),L_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{\mu}{1 + \psi} L_t^{1+\psi} \right),$$  \hspace{1cm} (1)

where $\beta$ is the discount factor, $\mu$ is the disutility of labor, and $\psi$ is the inverse Frisch-elasticity of labor supply. Aggregate consumption is $C_t = \left( \int C_t(i) (\theta - 1)/\theta \, di \right)^{1/\theta}$, where $C_t(i) = \left( \frac{1}{G} \sum_{g=1}^{G} [A_t(i,g)C_t(i,g)]^{(\gamma - 1)/\gamma} \right)^{\gamma/(\gamma - 1)}$. $C_t$ is a CES-aggregate of individual good consumptions $C_t(i,g)$ (with across-firm and across-good elasticity parameters $\theta$ and $\gamma$, respectively). The measure of firms $i$ is normalized to 1, and the number of goods per firm is $G$. $A_t(i,g)$ reflects the quality of the good, with higher quality goods providing larger marginal utility of consumption. These good- and producer-specific idiosyncratic shocks influence optimal prices, and generate dispersion that helps us to match the empirical size distribution of price changes.

The consumer’s budget constraint for each time period $t$ is given by

$$\int \sum_{g=1}^{G} P_t(i,g)C_t(i,g) \, di + B_{t+1}/R_t = B_t + W_tL_t + \tilde{\Pi}_t + T_t,$$  \hspace{1cm} (2)

where $P_t(i,g)$ is the nominal gross price of firm $i$ for product $g$, $B_{t+1}$ is a nominal bond with gross return $R_t$, $W_t$ is nominal wage, $\tilde{\Pi}_t$ is nominal profits, and $T_t$ is a lump-sum transfer.

The aggregate price level in this economy is $P_t = \left( \int P_t(i)^{1-\theta} \, di \right)^{1/\theta}$, where the producer-level aggregate price is $P_t(i) = \left( \frac{1}{G} \sum_{g=1}^{G} [P_t(i,g)/A_t(i,g)]^{1-\gamma} \right)^{1/(1-\gamma)}$. This implies that aggregate expenditure is given by $P_tC_t$. The representative consumer’s demand for each individual good $g$ of producer $i$ can be expressed as

$$C_t(i,g) = A_t(i,g)^{\gamma-1} \left( \frac{P_t(i,g)}{P_t(i)} \right)^{-\gamma} \left( \frac{P_t(i)}{P_t} \right)^{-\theta} C_t.$$  \hspace{1cm} (3)

The Euler-equation implies that $1/R_t = \beta E_t [P_tC_t/(P_{t+1}C_{t+1})]$. The labor supply equation is given by $\mu L_t^{\psi}C_t = W_t/P_t$. 

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2.3.2 The government and the central bank

In our baseline specification, we assume that money supply growth rate follows an autoregressive process with a drift

$$\log(M_t/M_{t-1}) = g_{Mt} = \mu_M + \rho_M g_{Mt-1} + \varepsilon_{Mt}$$

(4)

The growth rate of money supply introduces inflation to the model (with 0 technology growth, the inflation rate in the non-stochastic steady state equals $\pi = \mu_M/(1 - \rho_M)$). We assume that shocks to the money supply growth ($\varepsilon_{Mt}$) are expected to be 0 with probability 1. The extra money supply $M_t - M_{t-1}$ is redistributed to the consumer in a lump-sum way.

We assume that the value-added tax rate follows an exogenous unit root process

$$\tau_t = \tau_{t-1} + \varepsilon_{\tau t}.$$ 

We assume that shocks to the tax rates ($\varepsilon_{\tau t}$) are known with certainty $s$ periods before the change.

The net revenue of the firm is the fraction $1/(1 + \tau_t)$ of gross revenues, and the fraction $\tau_t/(1 + \tau_t)$ is paid for the government. These tax revenues of are also redistributed in a lump-sum way. Without loss of generality, we assume balanced budgets: $M_t - M_{t-1} + (\tau_t/(1 + \tau_t))P_tC_t = T_t$.

2.3.3 Firms

Each firm $i$ is assumed to sell its products $(i, g)$ ($g = 1, 2, \ldots, G$) in a monopolistically competitive market; post gross nominal prices $P_t(i, g)$ and satisfy all demand at these prices. Firms can adjust at least one of their prices ($P_t(i, 1), P_t(i, 2), \ldots, P_t(i, G)$) for a single fixed menu cost $\phi P_tC_t$. Firms maximize the expected discounted present value of their profits $E_0 \sum_{t=0}^{\infty} \frac{1}{\prod_{q=0}^{t} R_q} \tilde{\Pi}_t(i)$, where the periodic profit level (net of menu costs) is the difference between nominal revenues and production costs: $\tilde{\Pi}_t(i) = \sum_{g=1}^{G} [(1/(1 + \tau_t))P_t(i, g)Y_t(i, g) - W_tL_t(i, g)]$.

We assume that firms use a constant returns to scale technology with labor as the only factor. The production function is $Y_t(i, g) = L_t(i, g)/A_t(i, g)$ with higher quality products requiring extra labor input.\footnote{Similarly to Midrigan (2011), this assumption helps us to reduce the dimensionality of the problem.} We assume the log of the good quality follows a random walk\footnote{The results are robust to the case when the shocks follow an AR(1) process with persistence parameter $\rho_A = 0.7$: $\ln A_t(i) = \rho_A \ln A_{t-1}(i) + \varepsilon_t(i)$. Results are available from the authors upon request.}:

$$\ln A_t(i, g) = \ln A_{t-1}(i, g) + \varepsilon_t(i, g),$$

(5)

where innovations $\varepsilon_t(i, g)$ are mean-zero multivariate random innovations with variance $\sigma^2_A$ that are uncorrelated across firms, but they are correlated across goods $g$ within firms (Lach and Tsiddon, 1996; Fisher and Konieczny, 2000; Midrigan, 2011; Bhattarai and Schoenle, 2014). For the distribution of $\varepsilon_t(i, g)$, we choose a flexible parametric form, which generalizes the normally distributed innovations of Golosov and Lucas (2007) and the Poisson innovations of Midrigan (2011).
In particular, we assume that \( \varepsilon_t(i, g) \) is drawn from a mixture of two mean-zero multivariate normal distributions: with probability \( p \), the distribution has a (small) variance \(^{19}\) \( \lambda^2 \sigma^2 \), and with probability \( 1 - p \) it has a (large) variance \( \sigma^2 \). The cross correlations across goods \( g \) is \( \rho_\varepsilon \) in both cases.\(^{20}\)

\[
\varepsilon_t(i, g) = \begin{cases} 
N(0, \lambda^2 \sigma^2) & \text{with probability } p \\
N(0, \sigma^2) & \text{with probability } 1 - p
\end{cases}
\]

Our primary motivation for choosing this parametric form is that the proportionality between the standard deviations \( (0 \leq \lambda \leq 1) \) gives us a previously unexplored degree of freedom to influence the shape of the distribution with major impact of the extent of monetary non-neutrality. Nevertheless, the distribution is sensible in an environment where firms face firm-specific volatility shocks, which are not synchronized across firms.\(^{21}\)

The production function implies an individual and aggregate labor demand \( L_t(i, g) = A_t(i, g) \cdot Y_t(i, g) \) and \( L_t = \int \sum_{g=1}^{G} L_t(i, g) \, di \). We obtain a stationary period profit function (net of menu costs) by substituting labor demand and the households demand (equation (3)) into the periodic profit function \( \bar{\Pi}_t(i) \), using the equilibrium condition \( Y_t(i, g) = C_t(i, g) \) and the definition of the aggregate price index, and normalizing the resulting nominal profit function with the nominal GDP \( (P_t Y_t) \):

\[
\bar{\Pi}_t(i) = \sum_{g=1}^{G} \left[ \frac{1}{1 + \tau_t} p_t(i, g)^{1-\gamma} - w_t p_t(i, g)^{-\gamma} \right] \left( \frac{1}{G} \sum_{g=1}^{G} p_t(i, g)^{1-\gamma} \right)^{(\gamma - \theta)/(1 - \gamma)}. \tag{6}
\]

The variable \( p_t(i, g) = \frac{P_t(i, g)}{A_t(i, g)P_t} \) is the (good quality-adjusted) relative price of firm \( i \) of product \( g \), and \( w_t = W_t/P_t \) is the real wage. As it is apparent from equation (6), we can write the normalized profit function of the firm as \( \bar{\Pi}_t(p_t(i), w_t, \tau_t) \).

The firms’ decision on whether to adjust its prices depends on its last period relative prices and the current idiosyncratic shocks it faces. Our setup allows us to collapse these two idiosyncratic variables into a single state variable \( \mu_{t-1}(i, g) = \frac{P_{t-1}(i, g)}{A_{t-1}(i, g)P_{t-1}} = p_{t-1}(i, g) \frac{A_t}{A_{t-1}} \) for each firm \( i \) and good \( g \). It is its last period relative price deflated by the current good specific idiosyncratic shock. In our

\(^{19}\)To ensure that the variance of \( \varepsilon_t(i, g) \) is indeed \( \sigma_\varepsilon^2 \), we set \( \sigma^2 = \sigma_\varepsilon^2/(p\lambda^2 + 1 - p) \).

\(^{20}\)Our setup is different from that used by Midrigan (2011), who assumed that the product level shocks are constructed from independent shocks \( \tilde{\varepsilon}_t(i, g) \) following the formula \( \varepsilon_t(i, g) = \tilde{\varepsilon}_t(i, g) + \chi \text{mean}_p \{ \tilde{\varepsilon}_t(i, g) \} \), where \( \chi \) is a parameter influencing the cross-correlation \( \rho_\varepsilon \). The key difference is that in our case the extent of volatility is determined at the firm level, so if a firm’s first good is hit by a high (low) volatility shock, then its second good is also hit by a similarly high (low) volatility shock, while in Midrigan (2011) it is determined at the product level. Our main result is robust to this assumption: the mixed normal distribution leads to near monetary neutrality even if we used the formulation of Midrigan (2011). The results are available from the authors upon request.

\(^{21}\)We provide some suggestive evidence on the presence of such asynchronous idiosyncratic volatility in micro-level price data in Appendix D.
baseline specification, we assume no aggregate uncertainty about the aggregate variables, so our aggregate state in each period is the paths of the value-added tax rate ($\tau_t$), the level of the money supply $M_t$ and the cross-sectional distribution of adjusted relative price vectors $\Gamma_t$ known at time $t$, which we denote by $\Omega_t = \{\tau_t, M_t, \Gamma_t\}_{i=1}^\infty$.

In each period, after observing the idiosyncratic good quality innovations, firms decide to change nominal prices or keep them constant. If they change any of their prices, they pay the menu costs, which enable them to set their relative price vector optimally. If they decide not to change prices, their relative price vector gets equal to the one inherited from the previous period deflated by this period’s equilibrium inflation rate. The normalized value of the firm if it chooses not to change its prices is

$$V^{NC}(\mu_{t-1}(i), \Omega_t) = \overline{\Pi} \left( \frac{\mu_{t-1}(i)}{1 + \pi_t}, w_t, \tau_t \right) + \beta E_t V \left( \left\{ \mu_{t-1}(i,g) e^{-\varepsilon_{t+1}(i,g)} \right\}_{g=1}^G, \Omega_{t+1} \right), \quad (7)$$

where $\mu_{t-1}(i)$ is the vector of idiosyncratic states – the relative price vector inherited from the previous period and adjusted with this period’s good quality shock – at the time of the price change decision, and $e^{-\varepsilon_{t+1}(i,g)} = A_t(i,g)/A_{t+1}(i,g)$ is the inverse of the next period’s productivity innovation for each product $g$.

If the firm chooses to change its prices, it chooses the new relative price vector $p_t^*(i)$ optimally, and it pays menu costs, so its normalized value is

$$V^C(\Omega_t) = \max_{p_t^*(i)} \left\{ \Pi(p_t^*(i), w_t, \tau_t) - \phi + \beta E_t V \left( \left\{ p_t^*(i,g) e^{-\varepsilon_{t+1}(i,g)} \right\}_{g=1}^G \right), \Omega_{t+1} \right\}. \quad (8)$$

Finally, the value of the firm is determined by the upper envelope of $V^{NC}$ and $V^C$:

$$V(\mu_{t-1}(i), \Omega_t) = \max_{\{C, NC\}} \left[ V^{NC}(\mu_{t-1}(i), \Omega_t), V^C(\Omega_t) \right]. \quad (9)$$

---

22 This is different from Golosov and Lucas (2007) and Midrigan (2011) who calibrate their models taking aggregate volatility into consideration. The simplification has no qualitative impact on the results: their conclusions on aggregate monetary non-neutrality stay unaffected in a model without aggregate uncertainty, as we show later.

23 The relative price that was inherited from this period if there was no price change, $\mu_{t-1}(i,g)/(1 + \pi_t)$, needs to be adjusted with the next period’s productivity innovations $e^{-\varepsilon_{t+1}(i,g)} = A_t(i,g)/A_{t+1}(i,g)$, to obtain the relative price vector at the time of next period’s price change decision. The expectation is taken over the future idiosyncratic state variables, conditional on their current values.

24 The value function of the time-dependent pricing model of Calvo (1983) is

$$V(\mu_{t-1}(i), \Omega_t) = (1 - \kappa)V^{NC}(\mu_{t-1}(i), \Omega_t) + \kappa V^C(\Omega_t),$$

where the menu cost is set to zero $\phi = 0$ and $\kappa$ is now an exogenous probability of price change.
2.3.4 The equilibrium

In our baseline specification, we consider rational expectations equilibria without aggregate uncertainty. The equilibrium is a set of policy rules \( \{C_t(i,g), L_t, B_t\} \), price setting rules \( \{P_t(i,g)\} \), prices \( \{P_t, R_t, W_t\} \), policy shocks \( \{\tau_t, M_t\} \) and adjusted relative price distributions \( \{\Gamma_t\} \) for each \( t \), such that

1. The representative consumer’s policy functions \( \{C_t(i,g), L_t, B_t\} \) maximize her utility function \( (1) \) given her budget constraint \( (2) \),

2. The firms’ nominal price setting rule \( \{P_t(i,g)\} \) maximizes their normalized value functions \( (9), (7), (8) \) and they determine production \( \{Y_t(i,g)\} \) and labor demand \( \{L_t(i,g)\} \) to satisfy demand. The firms form correct beliefs about the random process of the idiosyncratic shocks \( \{A_t(i,g)\} \).

3. Money supply equals aggregate demand \( M_t = P_tC_t \) in each period \( t \). The seignorage revenue and the tax revenues are redistributed in a lump-sum way.

4. Goods markets \( C_t(i,g) = Y_t(i,g) \), bond market \( B_t = 0 \) and labor market \( \int \sum_{g=1}^{G} L_t(i,g) \text{d}i = L_t \) clear in each period \( t \).

5. The adjusted relative price distribution \( \{\Gamma_t\} \) develops consistently with the idiosyncratic shock distribution and the price setting rules.

We solve for this equilibrium numerically with standard global solution methods. We detail in Appendix B.3 our numerical solution algorithm for the steady state and the transition path after an unexpected, persistent money growth and pre-announced permanent tax shocks.

3 Calibration

We use conventional values for some basic parameters. We set the monthly discount factor to \( \beta = 0.96^{1/12} \), which implies a real interest rate of 4 percent. We set the inverse Frisch-elasticity of the labor supply (\( \psi \)) to zero, which implies a perfect partial wage-elasticity of labor supply. Under this assumption, nominal wages move in lockstep with the money supply, similarly to the models of Golosov and Lucas (2007) and Midrigan (2011). This condition also generates full long-term inflation pass-through of the value-added tax shocks that we also observe in the data. We set the growth rate of the money supply \( (\mu_M/(1 - \rho_M)) \) equal to the yearly observed trend inflation rate, which is 4.2 percent in our data.
We set the elasticity of substitution between products of different retail firms ($\theta$) to be $5^{25}$. In line with Midrigan (2011), we assume that each firm produces two goods ($G = 2$), and we set the elasticity of substitution for goods within a firm to be close to 1. We set the persistence of the idiosyncratic technology shock $\rho_A$ to unity, the value chosen by Gertler and Leahy (2008) and Midrigan (2011). We assume that the within-firm correlation between the idiosyncratic shocks are 60 percent that is also close to the calibration of Midrigan (2011).$^{26}$

We calibrate the menu cost and parameters that determine the shape of the idiosyncratic shock distribution to match key moments of the price-change distribution during months without tax changes. In our baseline model, we calibrate the menu cost parameter $\phi$ and the standard deviation of the idiosyncratic productivity shocks $\sigma_A$ to match the frequency and the average size of absolute price changes. We calibrate the probability of low variance shock $p$ to match the kurtosis of the price-change distribution, and the variance-proportionality parameter $\lambda$ to match the interquartile range of the absolute price-change distribution. The four moments exactly identify our four free parameters, and as the second column in the top panel of Table 2 shows we match exactly all our targeted moments.

We also calibrate special cases of our model with $\lambda = 0$ and $\lambda = 1$ for comparison. We do this by matching moments that are sufficient for their exact identification. For the Poisson distribution ($\lambda = 0$), we match the frequency, size, and kurtosis of the price-change distribution. For the Gaussian distribution ($\lambda = 1$), we match the frequency and the size of the price-change distribution. The last two columns of Table 2 show the steady-state moments of these models. The models with restricted $\lambda$ values do not match the untargeted moments well. The Gaussian distribution underestimates the kurtosis and the interquartile range. The Poisson distribution somewhat overestimates the interquartile range. Figure 2 compares the frequency of price changes of various size groups in the data with those implied by our baseline and restricted models. It confirms that our baseline model does a good job at matching the steady state price-change distribution. The figure suggests that it is only marginally better than the Poisson case, and they both match the observed price changes much better than the implied price changes of the model that is restricted to use the normal distribution. A consistent picture emerges from the behavior of the untargeted moments of the absolute-price-change distribution that we report in the bottom panel of Table 2.

Table 3 presents the calibrated parameters of our baseline model (mixed) and the special cases with normal and Poisson distributions. As Table 3 shows, our baseline parametrization implies a menu cost of 2.4 percent of steady-state revenues, paid with a 12.5%/2 probability (in a two-product framework, half of the price changes are free). So the overall cost of regular price adjustment is

$^{25}$This is an intermediate value between those used by Midrigan (2011) (3) and Golosov and Lucas (2007) (7). The value influences the estimates of menu costs and the standard deviation of idiosyncratic shocks without altering our conclusions.

$^{26}$In Section 5, we show that our results stay robust if we set this correlation to 1 or 0.
Table 2: Moments of regular price-change distribution and model moments

<table>
<thead>
<tr>
<th>Targeted moments</th>
<th>Data</th>
<th>Models</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Mixed</td>
<td>Poisson</td>
<td>Normal</td>
</tr>
<tr>
<td>Frequency</td>
<td>12.6%</td>
<td>12.6%</td>
<td>12.6%</td>
<td>12.6%</td>
</tr>
<tr>
<td>Absolute size</td>
<td>9.9%</td>
<td>9.9%</td>
<td>9.9%</td>
<td>9.9%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.98</td>
<td>3.98</td>
<td>3.98</td>
<td>1.97</td>
</tr>
<tr>
<td>Abs. interquartile range</td>
<td>8.13%</td>
<td>8.13%</td>
<td>9.55%</td>
<td>6.3%</td>
</tr>
<tr>
<td>Inflation</td>
<td>4.23%</td>
<td>4.23%</td>
<td>4.23%</td>
<td>4.23%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional moments (abs. size distribution)</th>
<th>Data</th>
<th>Models</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First decile</td>
<td>2.8%</td>
<td>2.8%</td>
<td>2.9%</td>
<td>3.6%</td>
</tr>
<tr>
<td>First quintile</td>
<td>4.6%</td>
<td>4.9%</td>
<td>4.4%</td>
<td>6.8%</td>
</tr>
<tr>
<td>Median</td>
<td>7.7%</td>
<td>7.5%</td>
<td>6.6%</td>
<td>10.1%</td>
</tr>
<tr>
<td>Third quintile</td>
<td>12.7%</td>
<td>13.1%</td>
<td>13.9%</td>
<td>13.1%</td>
</tr>
<tr>
<td>Ninth decile</td>
<td>19.4%</td>
<td>20.1%</td>
<td>21.6%</td>
<td>15.6%</td>
</tr>
</tbody>
</table>

The top panel of the table shows price-setting moments used for model calibration in the data and in models. In particular, it shows the frequency and kurtosis of regular price changes and the average size and interquartile range of the absolute value of the regular price changes. It shows the moments in models with our baseline mixed normal distribution as well as with the Poisson distribution ($\lambda = 0$), and normal distribution ($\lambda = 1$). The bottom panel of the table shows additional moments that are not used for model calibration.

Table 3: Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Menu cost</td>
<td>2.4%</td>
<td>1.6%</td>
<td>5.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Standard error</td>
<td>4.3%</td>
<td>4.4%</td>
<td>3.8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Probability of low volatility shock</td>
<td>91.2%</td>
<td>90.6%</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Relative standard deviation</td>
<td>8.8%</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table lists the calibrated parameters of the baseline model with the mixed normal distribution and contrast them with those of models using the Poisson and normal distributions.

around 0.15 percent of steady-state revenues. We consider it to be of a reasonable magnitude. Levy et al. (1997), for example, estimate the costs related to price adjustment in supermarkets to be 0.7 percent of revenues, which is higher than our estimates, but their measure includes the costs related to temporary sales that comprise the majority of observed price changes in supermarkets. Similarly to previous quantitative menu-cost models with idiosyncratic shocks, the model needs volatile idiosyncratic shocks to hit the large average absolute size of the price changes. The calibrated standard deviation is 4.3 percent (per month). The data favor a mixed normal idiosyncratic shock distribution where the low volatility shock is arriving with a high 91.2 percent probability and with
Figure 2: steady-state distribution of price changes

The figure plots the steady state price-change distributions in the data and in different menu-cost models. The baseline (mixed, $\lambda = 8.8$ percent) and the Poisson model ($\lambda = 0$) are similarly good at matching the observed distribution (see also Table 1), while the normal model ($\lambda = 1$) is less successful in matching the dispersion of the price changes.

a standard deviation that is only 8.8 percent of the high volatility shock.

Assuming a Poisson or a Gaussian distribution has a notable effect on the calibrated menu costs, but minor effects on the calibrated standard deviations.\textsuperscript{27} The Poisson distribution implies a probability of 0 idiosyncratic shock that is very close to our baseline (90.6 percent vs. 91.2 percent), but disallowing any variation in the low volatility shock requires a menu cost estimate that is 33 percent lower than our baseline. The reason for this is that smaller menu costs are sufficient to keep the frequency of steady state price changes on target. Not so with the Gaussian distribution, where the underestimated fraction of small price changes requires a menu cost that is more than twice as high as our baseline to keep the steady state frequency on target.

\textsuperscript{27}It should be noted that while these unconditional standard deviations are rather similar in the different menu-cost models, the conditional standard deviation of a non-zero shock in the Poisson case is much larger, $\sigma_A / \sqrt{1 - p} = 14\%$. Similarly, the productivity shock innovation has a standard deviation of $\sigma = \sigma_A / \sqrt{(p\lambda^2 + 1 - p)} = 14\%$ in the large standard-deviation case in the mixed normal model.
4 Large shocks

In this section, we confront our model with observations on pricing responses to large VAT shocks. The exercise is a quasi out-of-sample test because we have no remaining free parameters to alter predictions conditionally upon large shocks. We ask whether the predictions confirm our assumptions about the shape of the underlying shock distribution.

4.1 Inspecting the mechanism - a simplified model

The predictions of the model are sensitive to the shape of the idiosyncratic shock distribution. To gain some insight, we present a stylized menu cost model. We calibrate the model parameters to match steady state moments as we did in our full model. We use the model to explain how the details of the shock distribution change the magnitude of the frequency response and its asymmetry, as well as the shape of the cross-section of price changes after the tax shocks.

In this simplified model we consider an economy with a continuum of firms indexed by \( i \in [0, 1] \). Time is discrete \( t = 0, 1, \ldots \), and there are two subperiods within each period: day and night. During the day, firms choose either to keep constant the nominal price they inherited from the previous night \( (P_{it-1}) \), or to set a new price after paying a fixed menu cost \( \phi^2 \). During the night, firms face a frictionless environment when they can change their nominal prices for free. The existence of frictionless nights allows us to significantly simplify our analysis by dissecting the firms’ inherently dynamic price-setting problem into a series of static problems. As discussed earlier, we consider dynamic price setting in our full model.

We denote the firms’ optimal price by \( P^*_{it} \), and define the price gap as \( x_{it} = p_{it-1} - P^*_{it} \), where lower case letters denote logarithms. The firm faces a quadratic loss function \( (p_{it} - P^*_{it})^2 \) in both subperiods. This can be seen as a quadratic approximation of a more complex profit function. In the full model, we derived such a profit function from a standard constant elasticity of substitution demand.

We assume that at the beginning of each day, the price gap distribution takes a specific parametric form

\[
x_{it} \sim L(-\Delta m_t, \eta^2_{it}), \quad \eta^2_{it} = \begin{cases} 
\lambda^2 \sigma^2 & \text{with probability } p \\
\sigma^2 & \text{with probability } 1 - p 
\end{cases}
\]

where \( L \) denotes the Laplace distribution. This is an equilibrium outcome, if firms emerge from the night with zero price gaps\(^\text{28}\) and idiosyncratic shocks hit \( p^*_{it} \) with a Laplace distribution with \( L(0, \eta^2_{it}) \), and an unanticipated aggregate shock \( (\Delta m_t) \) modifies \( p^*_{it} \) of all firm \( i \) uniformly. The Laplace distribution is a combination of an exponential distribution and its mirror image for negative values.

\(^{28}\text{Under zero trend inflation } (\pi = 0) \text{ and unanticipated aggregate shocks, it is easy to see that all firms enter the days with an optimal price gap of zero. We will discuss below how this result changes with positive trend inflation.}\)
(double exponential, see Figure 3 below). Similarly to the normal distribution, it is a symmetric distribution with a location parameter \((-\Delta m_t)\) influencing its mean, and a scale parameter \((\eta^2_t)\) influencing its standard deviation. Differently from the normal distribution, the Laplace distribution has fat tails (its kurtosis is 6, while the kurtosis of the normal is 3). We use this parametric family of distributions to simplify the algebra; in the full model, we revert to the Gaussian distribution. The volatility of the distribution is either high \((\sigma^2)\) with probability \(1-p\), or low \((\lambda^2\sigma^2)\) with probability \(p\). In general (if \(\lambda \in (0,1)\)), the ergodic distribution of this process is a mixture of two Laplace distributions, and we will refer to this as the mixed-Laplace distribution. There are two interesting special cases: \(\lambda = 1\) implies volatility \(\sigma^2\) in each day, and the price gap distribution simplifies to the Laplace distribution. If \(\lambda = 0\), the price gap distribution becomes a “Poisson-Laplace”, where a Laplace shock arrives with a Poisson rate \(1-p\).

During the day, firms observe their price gaps and decide whether to pay the menu cost to adjust their prices or not. Firms know that they can adjust the prices for free during the night, so they only consider their single-period daytime profit. Their policy rule, thereby, is of a simple Ss type: they keep their prices constant in case \(|x_{it}| \leq \phi\), and set their price gap to 0 otherwise. Correspondingly, their adjustment hazard function \(\Lambda(\cdot)\) takes a simple form.

\[
\Lambda(x_{it}) = \begin{cases} 
0 & \text{if } |x_{it}| \leq \phi \\
1 & \text{otherwise}
\end{cases}
\]

Measurement of price changes is taking place during the day. The steady-state price-change distribution is simply the product of the adjustment hazard and the density of the price gap distribution \(l(x)\Lambda(x)\), where \(l\) denotes the density of the Laplace distribution with 0 mean. The exponential distribution has exponential tails, which simplifies the algebra and allows us to obtain closed-form solutions for key moments. As in our full model, we match four moments: the frequency and the kurtosis of price changes, and the size and interquartile range of the absolute price change distribution.\(^{29}\) We calibrate four parameters (the menu cost \(\phi^2\) and parameters of the idiosyncratic shock distribution: \(p, \sigma^2, \lambda\)) to match these moments.

The top left panel of Figure 3 plots the price-gap distribution (solid black line) and price change distribution (shaded areas) of our baseline model with the mixed Laplace distribution (\(\lambda = 0.15\)). The model is calibrated to match all four of our targeted moments. We compare the baseline model to two special cases: the bottom left panel shows the case when \(\lambda\) is restricted to be 1 (Laplace)\(^{30}\), and the right panel shows the case, when it is restricted to be 0 (Poisson-Laplace)\(^{31}\). The figures

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\(^{29}\)Section A1. of the Appendix contains the derivations of these moments.

\(^{30}\)In this case, menu costs and the volatility of idiosyncratic shocks are calibrated to match frequency and size of price changes.

\(^{31}\)In this case menu costs, the volatility of idiosyncratic shocks and the probability of a zero shock are calibrated to match frequency, size, and kurtosis of price changes.
illustrate two key characteristics of the role of the relative variance parameter ($\lambda$) in influencing the shape of the price-gap distribution and the size of the Ss bands: (i) as $\lambda$ declines the kurtosis of the price-gap distribution increases and with it the distribution’s peak becomes sharper and its tails become fatter, and (ii) the calibrated menu costs becomes lower and with it the inaction bands become narrower. The two observations are related: as more firms face smaller idiosyncratic shocks with higher probability, smaller menu costs are sufficient to keep a large number of firms inactive, as we observe it in the data.

Figure 3: Desired and actual price-change distributions
The figure plots desired and actual price-change distributions (black line and shaded areas, respectively) of the Laplace ($\lambda = 1$), Poisson-Laplace ($\lambda = 0$), and mixed-Laplace ($\lambda = 0.15$) distributions as predicted by the simple model. The parameters are calibrated to match key moments of the observed price-change distribution.

To indicate how positive trend inflation influences the position of the distribution, we introduce a fully anticipated aggregate shock of size $\pi > 0$ into the model. The aggregate shock hits at the beginning of each day and raises the desired price change of each firm. Firms anticipate this, so they optimally set a positive price gap ($x^* > 0$) during the night. As in Ball and Mankiw (1994)$^{32}$, the optimal initial price gap is strictly below half of the aggregate shock ($x^* < \pi/2$). Firms would set the gap at exactly $\pi/2$ if they could not adjust their prices during the day: They would set their prices for two periods (night and day) and $x^* = \pi/2$ would minimize their expected losses:

$^{32}$See the paper for an optimal closed form solution. As the paper shows, the optimal price gap depends on the shape of the idiosyncratic shock distribution. The variation caused by the shape, however, is quantitatively small, so we disregard this here. Our full model takes this effect into account.
halfway between their optimal price during the night and their expected optimal price during the
day. However, they can adjust their prices during the day for a fixed menu cost, and will do it if
they face a large enough idiosyncratic shock. They set their initial price gap strictly below \( \pi/2 \),
mostly because their pre-set price will stay effective only for smaller than average idiosyncratic
shocks. When the aggregate shock increases all firms’ desired prices by \( \pi \) during the day, the mean
of the desired price change distribution \(-x^* + \pi\) will be between \( \pi/2 \) and \( \pi \). This is depicted on
the left panel of Figure 4 for our baseline mixed-Laplace model. (We plot the distribution of desired
price changes \( dp^* = -x^* \), instead of the price gap distribution, because it is more intuitive to
think about the former than about the latter. In the previous sections and figures, when \( \pi = 0 \), the
two distributions are identical.) The anticipated aggregate shock pushes the desired price change
distribution to the right (black, solid line), and, consequently, the observed price change distribution
will be asymmetric with more price increases than price decreases.

![Figure 4: Impact of large shocks under positive trend inflation](image)

The figure plots desired and actual price-change distributions (black, solid line and shaded areas, respectively)
of the mixed-Laplace distributions under positive trend inflation in the steady state and their changes after
a large positive and a large negative aggregate shock.

We use the model now to demonstrate the impact of large unanticipated aggregate shocks. The
middle panel of Figure 4 shows the impact of a positive shock. It shifts the steady-state desired
price change distribution to the right. The shock modifies the actual price change distribution: it
adds new price increases (red shaded area) and removes previous price decreases (blue shaded area)
from the steady state actual price change distribution. The right panel of Figure 4 shows the impact
of a negative aggregate shock. It shifts the desired price change distribution to the left and modifies
the actual price change distribution: adds new price decreases (blue shaded area) and removes price
increases (red shaded area).
We can use the figures to gain some insights into how the shape of the product-level shock distribution influences the magnitude of the effects of large shocks. First, the large positive shock can have a substantial impact on the frequency of price changes. The reason is that the red area in the middle panel (new increases) is necessarily larger than the blue area (vanished decreases). The magnitude of the effect depends on the shape of the idiosyncratic shock distribution. In the baseline model, depicted here, the shock pushes the peak of the distribution outside of the inaction threshold, which raises the mass of increases substantially. The effect would be even larger in the Poisson-Laplace model (where the peak is even sharper), but smaller in the Laplace model (with smaller peak).

Second, the pass-through and the frequency effects of the positive versus negative large shocks are asymmetric. The asymmetry comes from the interaction of the unanticipated shocks and anticipated positive aggregate shock (trend inflation) that pushes the steady-state distribution to the right. As a result, a positive shock generates more price increases (red shaded area on the middle panel) than how much price decreases a negative shock generates (blue shaded area on the right panel) and a positive shock removes fewer price decreases (blue shaded area on the middle panel) than how much price increases a positive shock removes (right shaded area on the right panel). The extent of asymmetry depends on the shape of the distribution: it would be even higher in the Poisson-Laplace model (where the difference between the slope of the desired price change distribution around the positive and the negative inaction thresholds is even larger) and would be smaller in the Laplace model (where the same difference is smaller). Notably, we need large shocks to observe asymmetry (above the steady-state inflation): the effect of a marginal positive and a negative shock are symmetric even under positive trend inflation. The reason is that for marginal shocks the areas collapse to the densities above the inaction thresholds, and after a positive shock the density of new increases is the same as after a negative shock the density of vanished increases (both increasing the pass-through) and vice versa: after a positive shock the density of vanished decreases is the same as after a negative shock the density of new decreases (both increasing the pass-through).

Third, the average size of price changes can drop substantially when the large shock hits, if the idiosyncratic shock distribution has excess kurtosis, as in our baseline model. The reason is the shape of the distribution: most new actual price changes concentrate close to the inaction band as the large shock pushes the peak of the desired price change distribution outside the inaction band. A large fraction of these new price changes are smaller than the average steady-state price change, and this reduces the average in the month of the large shock.

Finally, the shape of the actual price change distribution also depends on the shape of the desired price change distribution when the large shock hits. When the large shock pushes a large fraction of the firms outside their inaction thresholds, the desired price change distribution temporarily reveals itself. Key moments, like the kurtosis and interquartile-range of the observed distribution, will
depend heavily on the kurtosis and interquartile range of the product-level shock distribution. This relationship is muted in case the aggregate shocks are small because then the menu cost prevents the translation of product-level shocks to actual price changes.

4.2 Predictions of the full model

We use the large tax changes described in Section 2 to test the predictions of our full model. For this, we take our baseline model, calibrated to match the steady-state price-change distribution, and hit it with permanent tax shocks that are announced before their implementation, as in reality.\textsuperscript{33} We do a similar experiment with models restricted to use the Poisson ($\lambda = 0$) and Gaussian ($\lambda = 1$) distributions and the Calvo (1983) model. The size of the shocks are calibrated to match the size of the tax changes, and the lags between the announcement and the tax changes are similarly calibrated to be in line with reality.

The second column of Table 4 presents the inflation pass-through ($\pi_t - \bar{\pi})/\Delta \tau_t$ observed in the data and the frequency of price changes of the affected products in the months with the tax changes. As discussed before, there was a substantial increase in the frequency of price changes within one month of the positive 3 and 5 percentage-point changes, and we observed large immediate inflation pass-through of the shocks (74 percent and 99 percent). Furthermore, the effects of the positive and negative shocks were asymmetric: the frequency response to a -5 percentage-point shock was only 27 percent (compared with the 62 percent to the similar sized positive shock), and its inflation effect was only 33 percent (compared with the 99 percent of the positive shock).

The third column of Table 4 confirms that our baseline model predicts the observed pricing effects of the tax shocks well. First, we come very close to hitting the inflation pass-through observed during the tax changing months (the model predicts 64, 88 and 27 percent versus the observed 74, 99 and 33 percents for the 3, 5 and -5 percentage-point shocks, respectively). The model achieves this mainly by matching the large frequency increases well, except during the 3 percentage-point shock, when it somewhat underestimates the frequency change. Second, the baseline model also

\textsuperscript{33}Disregarding preannouncement would not qualitatively change our conclusions, but it would have a sizable quantitative impact on the results. Its effect is particularly apparent in terms of the negative aggregate shocks: disregarding preannouncement would substantially increase the pass-through of the shock in the month of the tax decrease (to 48 percent in our baseline mixed normal case from the current 27 percent, while the impact of positive shocks would essentially be unchanged). What drives these results? It is predominantly related to the positive trend inflation that reduces the fraction of adjusting firms in the month of the negative shock. Firms foresee this, and as a result, they start adjusting their prices already before the shock hits by postponing costly price increases and reducing the magnitude of their price changes when they occur. In contrast, the preannouncement of a positive shock brings about less adjustment before the shock, because firms foresee that with high probability they are going to adjust in the month of the tax increase. Their reduced planning horizon, furthermore, might even limit their incentives to respond to idiosyncratic shocks (see Hobijn et al., 2006). Preannouncement in the Calvo model brings about a sizeable announcement effect (look at Figure 6) differently from the menu-cost models, because the probability of adjustment in the month of the shock is fixed at a low level, so firms start adjusting to it whenever they have the opportunity.
Table 4: Moments of price adjustment in the months with tax changes

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
<th>Calvo</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3% inflation</td>
<td>74%</td>
<td>64%</td>
<td>127%</td>
<td>41%</td>
<td>7.0%</td>
<td></td>
</tr>
<tr>
<td>+5% inflation</td>
<td>99%</td>
<td>88%</td>
<td>142%</td>
<td>49%</td>
<td>8.0%</td>
<td></td>
</tr>
<tr>
<td>-5% inflation</td>
<td>33%</td>
<td>27%</td>
<td>12%</td>
<td>39%</td>
<td>6.6%</td>
<td></td>
</tr>
<tr>
<td>Inflation pass</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>through</td>
<td>+3%</td>
<td>52%</td>
<td>32%</td>
<td>60%</td>
<td>18%</td>
<td>12.6%</td>
</tr>
<tr>
<td>through</td>
<td>+5%</td>
<td>62%</td>
<td>55%</td>
<td>90%</td>
<td>25%</td>
<td>12.6%</td>
</tr>
<tr>
<td>through</td>
<td>-5%</td>
<td>27%</td>
<td>19%</td>
<td>11%</td>
<td>17%</td>
<td>12.6%</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+3% frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+5% frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5% frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table lists the inflation pass through and the frequency of price changes in the data and the calibrated models after the 3-percentage-point and 5-percentage-point tax increases and a 5-percentage-point tax decrease.

generates substantial asymmetry in the inflation pass-through by predicting three times larger pass-through for the 5 percentage-point positive than for the similar size negative shock (88 percent vs. 27 percent), similarly to the data. We consider this all the more remarkable, as we do not have any free parameters to influence the behavior of the model when the shocks hit.

Alternative menu-cost models are unable to match the data moments, as columns 4-5 of Table 4 show. Using the Poisson distribution, we would substantially overestimate the frequency responses and the pass-through of the positive shocks, and also overestimate the asymmetry between the effects of similar sized positive and negative shocks. The normal distribution, in contrast, would underestimate the frequencies and pass-throughs of the positive shocks and would underestimate the asymmetry. Needless to say, the Calvo model (column 6) has no chance of matching the large frequency responses and the pass-throughs because of its hard-wired assumption of constant price-change frequency.

The distribution of price changes in the months with tax changes reveals valuable information about the unobserved idiosyncratic shock distribution. The shaded areas in Figure 5 show the realized distribution of non-zero price changes in the months with the tax changes. The histograms are ‘normalized’ to sum to the observed fraction of price changes during each particular tax-changing month. The observed distributions show excess kurtosis, each with a sharp peak and fat tails. The figures also show the predicted histograms calculated from our baseline model and the alternatives. The figures show that our baseline model is fairly successful in matching the distributions. The figures suggest, however, that the Poisson model overestimates, while the Gaussian model underestimates the kurtosis of the realized distributions.

The moments of the price-change distributions measured during the tax change months confirm these observations. The second column of Table 5 shows that the kurtosis of the price-change
Figure 5: Distribution of price changes in the months with tax changes. The figure plots the price-change distributions in the data and in different menu-cost models in the months with the tax changes. The baseline model (mixed) is successful in matching the frequency and the distribution of price changes. In contrast, the Poisson model overestimates the frequency and overestimates the kurtosis of the price-change distribution, and the normal model underestimates the frequency and the kurtosis (see also Table 5).

The distribution averaged around 9 in the months with the tax changes, increasing substantially from 4 observed during regular times. The interquartile range of the absolute price-change distribution during tax changes became tighter, averaging around 5.5 percent from the 8.2 percent observed normally. Interestingly, the average size of price changes decreased significantly during the tax changing months (less than 9 percent versus the 9.9 percent during regular times). All these observations are in line with the assumption of idiosyncratic shocks with excess kurtosis. The kurtosis increases, because a large fraction of the desired price-change distribution with high kurtosis gets revealed. The interquartile range becomes tighter for a similar reason: the range of the desired price-change distribution is much lower, but normally most of the small price changes are not realized because of the menu costs. As the aggregate shock deems these price changes worthwhile, the observed interquartile range collapses. The lower average price change is the consequence of the ample new below-average actual price changes. They are caused by the high relative mass of
firms around the center of the leptokurtic price gap distribution, which are pushed over the inaction thresholds.

The third column of Table 5 shows that our baseline model successfully predicts most of the moments. The kurtoses are reasonably close to those observed in the data, on average, even though, the model predicts significantly higher kurtosis for the 5 percentage-point positive than for the 5 percentage-point negative shock, contrary to the data. The interquartile ranges and the average size of price changes predicted by the model all matches the observed values quite closely. In contrast, the model with the Poisson distribution (the fourth column) would substantially overestimate the kurtoses and would underestimate the interquartile ranges and average sizes after the positive shocks. These results would reflect an underlying idiosyncratic shock distribution with a kurtosis that is too high. We observe the opposite when using the Gaussian distribution (the fifth column): it would underestimate the kurtoses and would overestimate the interquartile ranges and the average prices during tax-changing months. This suggests that the idiosyncratic distribution, in this case, would not have a sufficient kurtosis.

Table 5: Moments of price dispersion in the months with tax changes

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+5%</td>
<td>8.1</td>
<td>12.9</td>
<td>20.9</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td>9.2</td>
<td>6.0</td>
<td>3.5</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>+3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abs. interquartile range</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+3%</td>
<td>5.1%</td>
<td>4.7%</td>
<td>2.7%</td>
<td>6.5%</td>
<td></td>
</tr>
<tr>
<td>+5%</td>
<td>5.9%</td>
<td>4.3%</td>
<td>2.7%</td>
<td>6.6%</td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td>5.0%</td>
<td>5.7%</td>
<td>11.4%</td>
<td>6.6%</td>
<td></td>
</tr>
<tr>
<td>Absolute size</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+3%</td>
<td>6.5%</td>
<td>8.3%</td>
<td>6.8%</td>
<td>10.4%</td>
<td></td>
</tr>
<tr>
<td>+5%</td>
<td>9.0%</td>
<td>8.5%</td>
<td>7.8%</td>
<td>10.7%</td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td>8.6%</td>
<td>8.4%</td>
<td>10.2%</td>
<td>10.3%</td>
<td></td>
</tr>
</tbody>
</table>

The table lists the kurtosis of the regular price changes and the average size and interquartile range of the absolute regular price changes in the data and the calibrated models after the 3-percentage-point and 5-percentage-point tax increases and a 5-percentage-point tax decrease.

Figure 6 plots the dynamics of the monthly inflation rate in the affected groups of the processed food sector in the data, and presents simulated inflation paths in our baseline mixed normal model and in the alternatives. The baseline model captures the changes in the inflation rates and also broadly consistent with its dynamics. In contrast, the Poisson model would overestimate, and the normal model would underestimate the inflation effects of the shocks. In the Calvo model, very differently from the data and the menu-cost models, the inflation effect would peak right after the
The figures plot the monthly inflation rate in the processed food sector in the data and different menu-cost models. The baseline model (mixed) is successful in matching the changes in the inflation rates. In contrast, the Poisson model overestimates, and the normal model underestimates the inflation effects of the shocks (see also Table 4). In the Calvo model, very differently from the data and the menu-cost models the inflation effect peaks right after the announcement: the adjusting firms respond to the expected tax changes right away.

We conclude that the large and asymmetric pass-throughs of symmetric tax changes are in line with qualitative predictions of menu-cost pricing models. The Hungarian tax experiment provides
quasi out-of-sample support specifically to our baseline model with mixed normal product-level shock distribution. It captures the frequency and the inflation pass-through after these large aggregate shocks, as well as the observed distribution of price changes during tax-changing months.

5 Monetary non-neutrality

As we have shown in the previous section, our baseline model can capture key moments of the price-change frequency during normal times and also conditionally on large aggregate VAT shocks. In this section, we show that our model implies small and temporary real effects of standard monetary policy shocks.

To show this, we take our calibrated models and hit them with an unexpected, persistent money growth shock. We calibrate the persistence of the money shock ($\rho_M$) to be 0.61 and its standard deviation to 0.18 percent as in Midrigan (2011). We set the inflation rate to 0.34. Figure 7 presents the impulse responses.

We measure the pass-through as the proportion of the price-level effect of the shock over the cumulated change of the money supply, or formally as

$$\gamma_t = \frac{\sum_{i=0}^t (\pi_i - \bar{\pi})}{\sum_{i=0}^t \Delta m_i}.$$  \hspace{1cm} (10)

We measure the real effect as the cumulative output effect of the shock. The figure confirms that our baseline model implies small and temporary real effects (the cumulative real effects are the multiple of 1.59 of the monetary shock). It is close to the implications of the model using the Gaussian distribution (0.65). However, it predicts much lower real effects than the model with the Poisson distribution (7.60), which generates real effects close to that of the Calvo (1983) model (8.93). We show in Section 5.2 that these results are robust to the introduction of aggregate fluctuations (see Figure 9).

In the next section, we inspect the key mechanism behind this result. First, we assess the intuition in the simplified version of the model that we described in Section V.A; and then we will turn to the full quantitative model.

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34It should be noted that setting the inflation rate to zero for calibrations made at positive inflation rates is not equivalent to recalibrating the models to 0 inflation. The steady-state moments of the models change. The frequency, for example, in our baseline model becomes 11 percent; and it becomes 8.9 percent with Poisson and 12.4 percent with Gaussian shocks. We reset the price-change frequency parameter of the Calvo model to 8.9 percent, to maintain its comparability to the Poisson model. Our main conclusions are not affected, however, if we recalibrate the models to zero inflation.
The figure plots the impulse responses of the baseline multi-product model (mixed) and alternatives to a one standard deviation persistent money growth shock. The figure shows that the baseline model implies small and temporary real effects (the cumulative real effects are the multiple of 1.59 of the monetary shock). They are substantially lower than the Calvo (1983) (8.93) and the Poisson model (7.60), and only somewhat larger than the model with the normal distribution (0.65).

5.1 Inspecting the mechanism

Let us now consider the simplified model of Section V.A again without trend inflation $\pi = 0$, to gain some insights on the driving forces behind the impact of the shape of the product-level shock distribution on the extent of monetary non-neutrality. It is instructive to decompose the aggregate price effect of a marginal aggregate shock $\Delta m_t$ following Caballero and Engel (2007). The change in the aggregate price level in the model equals $\Delta P_t = \int -x_{it}\Lambda(x_{it})l(x_{it})dx_{it}$, consequently, the price level impact of a marginal aggregate shock is

$$
\gamma_t = \left. \frac{\Delta P_t}{\Delta m_t} \right|_{\Delta m_t \to 0} = \int \Lambda(x_{it})l(x_{it})dx_{it} + 2\phi l'(\phi).
$$

The expression shows that the price-level impact of a marginal aggregate shock can be decomposed into two components. The intensive margin reflects the impact of the higher price adjustment of firms that would have changed their prices even without the aggregate shock. They adjust the size of their price change to fully accommodate the impact of the aggregate shock on their optimal prices. The aggregate impact from the intensive margin effect, therefore, equals the steady-state frequency of price changes. The price-level effect of the monetary shock is also influenced by another
component, the selection of two sets of firms that are on the verge of price adjustment. Firms with negative price gaps which are indifferent between adjusting their prices (at their inaction threshold) without the aggregate shock now find it optimal to adjust. They increase their prices by half of the width of their inaction band, \( \phi \). The density of these adjusters at the inaction threshold is \( l_1(\phi) \). Firms with positive price gaps at the inaction threshold cancel their adjustment as a response to the aggregate shock. They would have decreased their prices by \( \phi \), and the density of these non-adjusters at the inaction threshold is also \( l_2(\phi) \). The impact of the selection on the aggregate price level after a marginal shock, therefore, depends on the product of the width of the inaction band \( 2\phi \) and the density at the inaction thresholds \( l(\phi) \). As we show in a moment, both of these factors are influenced by the shape of the price gap distribution, so varying the volatility ratio \( \lambda \) has a non-linear impact on the selection effect.

Our question is how the shape of the price gap distribution influences monetary non-neutrality in our menu-cost model. The volatility ratio \( \lambda \) determines the shape of the distribution. We, therefore, ask, how changing the value of this parameter between 0 and 1 influences the pass-through of an infinitesimal monetary shock \( \gamma_t \). In the process, we use information inherent in the observed price-change distribution. In particular, we calibrate the other key parameters of the model: the menu cost parameter \( \phi \), the probability of a low volatility shock \( p \) and the standard deviation of the large volatility shock \( \sigma \) so as to match three standard moments of the steady-state price-change distribution in the Hungarian data: the frequency (12.6 percent), the average absolute size (9.9 percent) and the kurtosis (4) of price changes. Figure 3 in Section V.A plots the resulting price-gap and price change distributions for \( \lambda = 0, 0.15, 1 \).

We can see the impact of changing parameter \( \lambda \) more clearly in Figure 8. The Figure plots the selection effect, the width of the inaction band and the densities at the inaction thresholds for different \( \lambda \) values. For each \( \lambda \), other parameters of the model are re-calibrated to match the frequency, absolute size and kurtosis observed in the data. It is sufficient to concentrate on the selection effect because this drives the differences between the pass-throughs across the different models. The reason for this is that the intensive-margin effect equals the frequency of steady-state price changes, which is kept constant. We can match all three moments only for a subset of \( \lambda \) values, and only those are plotted on the x-axis. For comparison, the figures also show the values obtained for \( \lambda = 1 \) case with an unmatched kurtosis. The figure shows that a relatively minor deviation from the Poisson-Laplace case implies a sizable increase of the selection effect. The reason for this is that a lower excess kurtosis (or higher \( \lambda \)) increases both the width of the inaction band and the densities at the thresholds, driving the selection effect quickly close to levels observed with the simple Laplace case. The price-level effect of a monetary policy shock, therefore, quickly becomes over 50 percent, which implies high price-level flexibility. Notably, for an overwhelming fraction of the parameter space, except very close to \( \lambda = 0 \) case, the model predicts near neutrality of money.
Figure 8: Selection and determinants
The figure plots the selection effect and two of its determinants, the width of the inaction band and the density at the inaction thresholds, as a function of the volatility-ratio parameter ($\lambda$). Other parameters of the model are re-calibrated to match key steady-state price-setting moments. The figure shows that the selection effect is very sensitive to small deviations from the Poisson-Laplace case ($\lambda = 0$). The model predicts high selection, and, therefore, money near-neutrality for most of the relevant parameter space.

This result is not inconsistent with the conclusion of Alvarez et al. (2016). They also find that the extent of monetary non-neutrality is unidentified by the kurtosis and the frequency of price changes – sufficient statistics in a large class of models – if the cost distribution has fat-tails with infrequent and large jumps, as in our case. This result is also related to Elsby and Michaels (2014), who also find that the shape of the idiosyncratic shock distribution is critical in terms of monetary neutrality in menu-cost models. They find that a continuous idiosyncratic shock distribution (like the Laplace or mixed-Laplace) implies monetary near-neutrality as the menu cost approaches 0, but discontinuous distributions with an atom (like the Poisson-Laplace distribution) can imply money non-neutrality not dissimilar from the Calvo (1983) model even as the menu cost approaches 0. Our example shows that for realistic menu costs away from 0, the distinction between continuous and discontinuous distributions are not so clear-cut. In particular, the Poisson distribution is not a knife-edge case, and a whole range of continuous idiosyncratic shock distributions around it with positive, but still fairly small menu costs calibrated to match the steady-state price-change frequencies generate money non-neutrality similar to Calvo (1983). The reason is that these distributions require higher menu costs to match the observed price change frequencies and the costs will counteract the impact of the more dispersed idiosyncratic shock distributions. So ultimately choosing between relevant idiosyncratic shock distributions is an empirical question, and our result points towards the validity of a continuous distribution with excess kurtosis.
5.2 Robustness

In this section, we analyze whether the mixed normal distribution similarly leads to near monetary neutrality in various modifications of the model. We vary the models across two dimensions. First, we introduce aggregate fluctuations. The presence of aggregate shocks change the calibrated parameters, like the size of the menu cost, so they might affect the predictions of the model. Second, we introduce stochastic menu costs, which can reduce selection and increase monetary non-neutrality, as argued by Dotsey et al. (1999). We show that for realistic parameter values, our conclusions are robust to these modifications.

5.2.1 Aggregate fluctuations

We introduce aggregate fluctuations through the framework proposed by Krusell and Smith (1998). The key insight of the framework is that even though the state space is infinite dimensional (includes the distribution of relative prices), the firms’ and households’ policy functions depend on one key moment – the real wage, in our case. The real wage, in turn, can be well approximated as a function of the previous period real wage and the current period money growth shock (the sole source of aggregate fluctuations in our exercise), so we assume that agents form their expectations about real wage accordingly \( w_t^e = f(w_{t-1}, g_{Mt}). \)

In our baseline specification, the estimated goodness of fit \( R^2 \) of this equation on simulated data is higher than 0.999. Therefore, the framework approximates the rational expectations equilibrium well.

We calibrate models with aggregate fluctuations under zero trend inflation. (Remember that in our baseline, we calibrated parameters under positive inflation, and set inflation to zero to simulate monetary policy shocks.) This way our robustness test is more stringent: we not only introduce aggregate fluctuations but also eliminate trend inflation. Trend inflation and aggregate fluctuations have a similar impact on the size of the calibrated menu cost. Inflation creates a drift in relative prices, which drives firms constantly towards their inaction thresholds. This raises

\[ \text{In particular, we assume that agents use the following equation to form expectations about real wages: } \ln w_t^e = \alpha + \alpha_u \ln w_{t-1} + \alpha_m g_{Mt} + \alpha_m^2 100g_{Mt}^2 + \alpha_m^3 10000g_{Mt}^3. \]

We include higher order terms of the money shock to allow for asymmetry and non-linear effects of shock sizes. Table 8 in Appendix B.5 shows some evidence for asymmetry: the real effects of negative shocks are larger than positive shocks, but the asymmetry is small for realistic monetary policy shocks. Inflation expectations are derived from the real wage expectations, consistently with the model’s equilibrium conditions: in particular the labor supply equation, which requires the equality of the growth rates of money and nominal wages \( g_{Mt} = \ln W_t - \ln W_{t-1} \). Real wages are defined as \( \ln w_t = \ln W_t - \ln P_t \), which implies that \( \pi_t = g_{Mt} - (\ln w_t^e - \ln w_{t-1}) \). The equilibrium with aggregate uncertainty requires that both households and firms maximize their utility and profits given the expectation function above, and the general equilibrium behavior of the model is consistent with the estimated parameters of the expectation function.

\[ \text{This is true for both our baseline mixed normal distribution and for the normal distribution. The } R^2 \text{ is 0.98, still acceptable, when we use the Poisson distribution.} \]

\[ \text{Table 7 in the appendix shows that the change in the parameters is smaller, so the predictions are even closer to the baseline if we calibrate the model with both positive inflation and aggregate fluctuations.} \]
the frequency of price changes, which needs to be counterbalanced by calibrating higher menu costs. Similarly, aggregate fluctuations vary relative prices, which regularly push firms outside their inaction thresholds. This raises the frequency and also requires a higher menu cost. Our exercises show that aggregate fluctuations can ‘substitute’ trend inflation in the sense that we can match steady state moments under aggregate fluctuations even without trend inflation. This allows us to examine the robustness of our results among a wider range of realistic parameter values.

![Figure 9: Impulse responses to a monetary shock](image)
The figure plots the real effects of small (one standard deviation) monetary shock. The first row shows models with multiple products, and the second row models with heterogeneous menu costs. Models in the left column disregard aggregate fluctuations, models on the right include them. The figure shows that our benchmark results are robust.

The top right panel in Figure 9 shows the output effect of a monetary policy shock in a multi-product setup with aggregate fluctuations.\(^{38}\) The predicted impact of the model with the mixed normal distribution (black, solid line) is virtually unchanged relative to our baseline. As before, the real effects are small and temporary and closer to the case with the normal distribution (blue dotted line), than to the case with the Poisson distribution (red dashed line) and the Calvo model (green line). The impulse responses show the average difference between a scenario with a one standard deviation money growth shock relative to an alternative scenario with no money shock; both following the same long random sequence of monetary policy shocks. Therefore, the impulse responses take into account the impact of the initial position of the price distribution when the shock hits as well as the uncertainty about the future monetary policy shocks.
These results confirm that realistic aggregate fluctuations and trend inflation are both small, so they play a quantitatively marginal role in influencing the calibrated parameters and, therefore, the extent of monetary non-neutrality—instead non-neutrality is predominantly driven by the shape of the idiosyncratic shock distribution, as in our baseline specification.

Table 6: Moments of regular price-change distribution and model moments with aggregate fluctuations and zero inflation

<table>
<thead>
<tr>
<th>Targeted moments</th>
<th>Data</th>
<th>Models</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mixed</td>
<td>Poisson</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>12.6%</td>
<td>12.6%</td>
<td>12.6%</td>
<td>12.6%</td>
</tr>
<tr>
<td>Absolute size</td>
<td>9.9%</td>
<td>9.9%</td>
<td>9.9%</td>
<td>9.9%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.98</td>
<td>3.98</td>
<td>3.98</td>
<td>1.63</td>
</tr>
<tr>
<td>Abs. interquartile range</td>
<td>8.13%</td>
<td>8.13%</td>
<td>11.72%</td>
<td>6.41%</td>
</tr>
<tr>
<td>Inflation</td>
<td>4.23%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional moments (abs. size distribution)</th>
<th>Data</th>
<th>Models</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First decile</td>
<td>2.8%</td>
<td>2.5%</td>
<td>1.6%</td>
<td>3.5%</td>
</tr>
<tr>
<td>First quintile</td>
<td>4.6%</td>
<td>4.7%</td>
<td>4.4%</td>
<td>6.8%</td>
</tr>
<tr>
<td>Median</td>
<td>7.7%</td>
<td>7.4%</td>
<td>6.4%</td>
<td>10.1%</td>
</tr>
<tr>
<td>Third quintile</td>
<td>12.7%</td>
<td>12.8%</td>
<td>14.8%</td>
<td>13.2%</td>
</tr>
<tr>
<td>Ninth decile</td>
<td>19.4%</td>
<td>21.5%</td>
<td>23.2%</td>
<td>15.8%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output volatility</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-product</td>
<td>0.16%</td>
<td>0.27%</td>
<td>0.11%</td>
</tr>
<tr>
<td>Random menu cost</td>
<td>0.16%</td>
<td>0.25%</td>
<td>0.10%</td>
</tr>
</tbody>
</table>

The top panel of the table shows price-setting moments used for model calibration in the data and in models. In particular, it shows the frequency and kurtosis of regular price changes and the average size and interquartile range of the absolute value of the regular price changes. It shows the moments in models with our baseline mixed normal distribution as well as with the Poisson distribution ($\lambda = 0$), and normal distribution ($\lambda = 1$). The middle panel of the table shows additional moments that are not used for model calibration. The bottom panel shows the volatility of real output in the two model versions where we have aggregate fluctuations. The output volatility in the Calvo model is 0.40%.

In Table 6 we report the targeted and untargeted moments in the model with aggregate fluctuations and zero inflation (the model that generates the impulse responses in the top right panel of Figure 9). The top panel shows the targeted moments, and the middle panel of the table shows the deciles and the quartiles of the size distribution of absolute price changes, both in data and in the three models. Our preferred model with mixed normal idiosyncratic shock distribution performs well, as the unmatched moments are quite close to moments calculated from data. The two alternative models do not match these moments so closely: the Poisson model generates too many small price changes, while the Gaussian model generates too few small price changes. As discussed, this

35
is because in the model with Poisson shocks, the calibrated menu cost is too small and the inaction band is too narrow, while in the model with Gaussian shocks the calibrated menu cost is too large and the inaction band is too wide.

The bottom panel of Table 6 provides additional information about the standard deviation of output in the different models, which is a relevant statistic of the degree of monetary non-neutrality. These results are in line with the impulse responses: in the model with Gaussian shocks, the standard deviation of output is small (0.11%), and the model with mixed normal idiosyncratic productivity shocks comes close to this (0.16%). However, the model with Poisson shocks produces a much higher output standard deviation (0.27%), which is close to the figure calculated from the Calvo model (0.40%).

5.2.2 Stochastic menu costs

Stochastic menu cost can lead to higher monetary non-neutrality than the deterministic menu cost we use in our baseline (Dotsey et al., 1999). An intuition behind this result is that stochastic menu cost introduces ‘calvoness’ to the model (Villar and Luo, 2017) – indeed, the Calvo (1983) model is equivalent to a particular stochastic menu-cost model, where menu cost can be either zero or infinity, and the probability of the former is $1 - \lambda$ and iid over time. Realistic probability distributions of stochastic menu costs tend to imply high calvoness, with a sizable fraction of firms that faces tiny costs, while another large fraction that faces large costs. This implies low selection and high monetary non-neutrality, as in the Calvo (1983) model. However, calvoness has another effect: it limits the flexibility of prices after a large shock: if a large fraction of firms face large menu costs, they will not adjust even if the shock is large (in the limiting Calvo (1983) model the frequency even stays constant). Observations after large shocks, therefore, can help us assess the amount of calvoness in a stochastic menu cost model that we can consider realistic. This is what we do in this section.

We introduce a simple stochastic menu cost into the single-product version of our model. The (standardized) menu costs can take two values: they are either 0 with probability $\kappa$, or they are $\phi$ with probability $1 - \kappa$, and the distribution is iid over time. In other words, a constant fraction of the firms can change their prices for free, and the remaining fraction needs to pay a constant menu cost. The framework introduces a new free parameter $\kappa$, and it is straightforward to see that the parameter influences the calvoness of the model, and with it both the flexibility of prices after a large shock and monetary non-neutrality. If $\kappa$ is 0, the model collapses to the limiting single-product menu cost case. In this case, both monetary non-neutrality and flexibility after large shocks are determined by the shape of the idiosyncratic shock distribution, as we discussed

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39The predictions of the single-product and the baseline multi-product model are close both during normal times and after large shocks.
above. At the other extreme, as $\kappa$ approaches the steady-state frequency of price changes, which the model needs to match, $\phi$ needs to increase without limit; otherwise the frequency would be overestimated. However, when $\kappa$ equals the frequency, and $\phi$ is infinity we are exactly in the Calvo (1983) model, which features no selection and no flexibility after large shocks. We use the frequency of price changes observed in Hungary after the VAT shock to calibrate this parameter and assess the amount of monetary non-neutrality as predicted by the model.

The calibration exercise is analogous to the one we did in our baseline, except now we have one extra parameter: $\kappa$ and one extra moment: the frequency of price changes in the month of the five percentage points tax increase. We use the four parameters of the mixed normal idiosyncratic shock distribution to match the frequency, size, kurtosis and inter-quartile range of the price change distribution during normal times as before. We disregard aggregate fluctuations. We find that a small value $\kappa = 0.81\%$ is consistent with the observations. This value keeps the model very close to the menu cost model and far from the Calvo (1983) model (remember, that $\kappa = 12.6\%$ would be equivalent to it). As the model stays close to the menu cost model, its predictions about monetary non-neutrality also remain robust. As the black, solid line on the lower left panel of Figure 9 shows, the impulse responses after a monetary policy shock are small and temporary as in our baseline. As the black, solid line on the right panel shows, the results stay robust if we introduce aggregate fluctuations to the model and set inflation to zero, as we did in our previous robustness exercise. The figures also show that for this $\kappa$ parameter, varying the shape of the idiosyncratic shock distribution has a similar impact as in our baseline: the Poisson distribution ($\lambda = 0$) leads to sizable real effects, while normal distribution ($\lambda = 1$) leads to small and temporary real effects.

6 Conclusion

We have presented novel facts about pricing responses to large aggregate shocks and developed a menu-cost model that fits these observations. The observations are informative about key unobserved features, like the shape of the underlying shock distribution. We have shown that this shape is critical in determining the magnitude of the monetary non-neutrality in the model. The distribution favored by the data predicts small and temporary real effects of monetary policy shocks.

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40 With a Poisson distribution, a high $\kappa = 4.3\%$ would match the frequency of price changes after the positive VAT shock. This model would still fare worse than the mixed normal model in matching moments of the price change distribution. It would underestimate the interquartile range during normal times and overestimate the asymmetry and lead to excessively centered distribution (with too low interquartile range and too high kurtosis) after large shocks. The money non-neutrality in this model would be high, actually higher than the Poisson model shown in the lower left panel ($\kappa = 0.81\%$) and close to the impulse responses in the multi-product setting.

41 These results on the non-neutrality of money across distributions stay robust to a much higher value of $\kappa = 6.3\%$, i.e., when we move halfway from a simple menu cost model towards the Calvo (1983) model. These results are available from the authors upon request.
Our conclusion raises some doubts about the adequacy of menu cost models with realistic idiosyncratic shock distribution to explain the rigidity of the price level observed in time-series evidence (Bernanke and Blinder, 1992; Gertler and Karadi, 2015). One might require additional frictions, like wage rigidities, real rigidities or information frictions to overcome this fundamental micro-macro dichotomy. These additional rigidities, however, should not affect the model’s ability to match the price-change distribution and generate flexible and asymmetric responses to large aggregate shocks, consistent with the new evidence presented in our paper. We leave this for further research.

References


Online Appendix

A  Simple model

A.1  Moment of the price change distribution

In this section, we derive the key moments of the price-change distribution in the simple model. In particular, we derive the frequency ($I$) and kurtosis ($K$) of the price changes and the average size ($\Delta P$) and interquartile range ($IQR$) of the absolute price change distribution. We derive the moments for the case with no stochastic volatility $\lambda = 1$. In this case, the price gap distribution is Laplace (double exponential) with a scale parameter $\sigma$. The density is

$$l(x) = \begin{cases} \frac{1}{2\sigma}e^{-\frac{x}{\sigma}} & \text{if } x \geq 0 \\ \frac{1}{2\sigma}e^{\frac{x}{\sigma}} & \text{if } x < 0 \end{cases}$$

Quadratic loss function $x^2$ and a menu cost of $\phi^2$ implies that the inaction thresholds are given by $\pm \phi$, so the adjustment hazard is

$$\Lambda(x) = \begin{cases} 0 & \text{if } |x| \leq \phi \\ 1 & \text{otherwise} \end{cases}$$

The frequency of price changes is determined by the measure price gaps outside the inaction thresholds

$$I = \int_{\phi}^{\infty} \frac{1}{\sigma} e^{-\frac{x}{\sigma}} dx = e^{-\frac{\phi}{\sigma}}$$

The price-change distribution $dp(x) = l(x)\Lambda(x)/I$ is a combination of tails of an exponential and its mirror image to the vertical axis.

$$dp(x) = \begin{cases} \frac{1}{2\sigma} \left(\frac{1}{\sigma} e^{-\frac{x}{\sigma}} + \frac{1}{\sigma} e^{\frac{x}{\sigma}}\right) & \text{if } x \geq \phi \\ \frac{1}{2\sigma} \left(\frac{1}{\sigma} e^{\frac{x}{\sigma}} - \frac{1}{\sigma} e^{-\frac{x}{\sigma}}\right) & \text{if } x < -\phi \end{cases}$$

The kurtosis of the price-change distribution is

$$K = \frac{E[|x|^4]}{(E[|x|^2])^2} = \frac{2 \int_{\phi}^{\infty} x^4 \frac{1}{2\sigma} \frac{1}{\sigma} e^{-\frac{x}{\sigma}} dx}{\left(2 \int_{\phi}^{\infty} x^2 \frac{1}{2\sigma} \frac{1}{\sigma} e^{-\frac{x}{\sigma}} dx\right)^2} = \frac{I^4 + 6\sigma^2 I^2 + 8\sigma^3 I + 9\sigma^4}{I^4 + 2\sigma^2 I^2 + \sigma^4},$$

which we obtained by repeated application of integration by parts and some straightforward algebra. We also used the symmetry of the distribution.
The distribution of the absolute price changes is a tail of an exponential distribution.

\[ \text{adp}(x) = \frac{1}{e^{-\frac{x}{\sigma}}} e^{-\frac{x}{\sigma}} \text{ if } x \geq \phi \]

The average size of absolute price changes is the mean of this distribution

\[ \Delta P = \int_\phi^\infty x \frac{1}{e^{-\frac{x}{\sigma}}} e^{-\frac{x}{\sigma}} \, dx = \phi + \sigma \]

The first quartile \((Q_1)\) of the absolute price change distribution is implicitly determined by

\[ \frac{1}{4} = \int_{\phi}^{Q_1} \frac{1}{e^{-\frac{x}{\sigma}}} e^{-\frac{x}{\sigma}} \, dx = 1 - e^{-\frac{Q_1 - \phi}{\sigma}}, \]

from which

\[ Q_1 = \phi - \sigma \log \frac{3}{4}. \]

Analogously

\[ Q_3 = \phi - \sigma \log \frac{1}{4}, \]

from which

\[ IQR = Q_3 - Q_1 = \sigma \log 3. \]

If \(\lambda = 0\), the price-gap distribution is a Pareto-Laplace with a probability distribution

\[ PL(x) = \begin{cases} 
    \frac{1-p}{2} e^{\frac{x}{\sigma}} + p & \text{if } x < 0 \\
    \frac{1-p}{2} e^{-\frac{x}{\sigma}} + p + \frac{1-p}{2} & \text{if } x = 0 \\
    -\frac{1-p}{2} e^{\frac{x}{\sigma}} - p + \frac{1-p}{2} e^{-\frac{x}{\sigma}} - p & \text{if } x > 0
\end{cases} \]

with a mass point at \(x = 0\).

If \(\lambda \in (0, 1)\), the price gap distribution is a mixed Laplace with a density

\[ ml(x) = \begin{cases} 
    \frac{1-p}{2\lambda} e^{\frac{x}{\sigma}} + \frac{p}{2\lambda} e^{-\frac{x}{\sigma}} & \text{if } x \geq 0 \\
    \frac{1-p}{2\lambda} e^{-\frac{x}{\sigma}} + \frac{p}{2\lambda} e^{\frac{x}{\sigma}} & \text{if } x < 0
\end{cases} \]

For cases \(\lambda \in [0, 1)\), the moments can be obtained analogously to the \(\lambda = 1\) case, and the derivations are available from the authors on request.
B Full model

B.1 Equivalence of inflation effects of money and tax shocks

In this section, we show that for our baseline parametrization, permanent money shocks and value-added tax shocks have equivalent effects on the inflation path - even though their effects on the output is different. The proof guesses the same price level paths and verifies that the money supply and the tax rate have equivalent effects on the optimal price choices, justifying the equal-price-effect assumption.

Nominal wage moves together with the money supply, under our assumptions on the labor supply equation with separable utility that is logarithmic in consumption and linear in labor \((\psi = 0)\). In this case

\[
W_t = \mu P_t Y_t = \mu M_t.
\]  

(11)

Substituting this into equation (6) about the periodic normalized profit, one can easily re-write the profit equation as

\[
\bar{\Pi}_t(i) = \left( \frac{1}{1 + \tau_t} \right) \sum_{g=1}^{G} \left[ p_t(i, g)^{1-\gamma} - \frac{\mu M_t (1 + \tau_t)}{P_t} p_t(i, g)^{-\gamma} \right] \left( \frac{1}{G} \sum_{g=1}^{G} p_t(i, g)^{1-\gamma} \right)^{(\gamma-\theta)/(1-\gamma)}.
\]

(12)

Let’s guess that the present and future path of the price level \(\{P_t\}\) is the same for a permanent tax shock and a permanent money level shock. The optimal price choices of firms depend on their normalized value function, which is a present discounted value of their future profits. As the derivation shows, the tax rate influences the level of profits, but its influence on the optimal price choice is equivalent to that of the money supply.\(^{42}\) As we also assume lump-sum redistribution of taxes, the variables will not influence the budget constraints either. It means that the assumption of equivalent price level development is indeed verified. So we are justified to use evidence gained from value-added tax shocks to test the predictions of our model to large permanent money shocks.

B.2 The flexible price equilibrium

The algebraic solution for the flexible price equilibrium provides useful information about the long-term pass-through of the permanent tax- and money shocks. Money shocks, naturally, have no real effects under flexible prices, so we will have full pass-through to the price level. A permanent value-added tax shock, for our parametrization, will imply a unit drop in the real output, so under

\(^{42}\)For that to be exactly true, we need to assume that menu costs are tax deductible, so their effective costs drop with higher value-added taxes together with the value functions.
constant money supply, we will have full pass-through into the gross nominal prices in this case as well. 43

To gain some insight into why value-added tax shocks imply a unit drop in output, it is useful to look at the firms’ static profit maximization problem. Under flexible prices, firms will choose prices to maximize this, implying the following optimal relative price:

\[ p_{t,i,g}^* = (1 + \tau_t) \frac{\gamma}{\gamma - 1} w_t, \]  

where \( w_t = W_t/P_t \) is the real wage. The equation shows that each firm want to increase their relative prices as a response to a tax increase. As all firms cannot do this in equilibrium, real wages have to drop endogenously. It requires lower labor demand and output; and as household wage income will drop in parallel, the aggregate demand will adjust sufficiently to satisfy general equilibrium.

As all firms will choose the same productivity-adjusted relative price, the Dixit-Stiglitz-aggregate of these relative prices – that needs to be equal to 1 by definition – is also \( \gamma w_t (1 + \tau_t)/((\gamma - 1) = 1. \)

We find that

\[ w_t = \frac{\gamma - 1}{\gamma (1 + \tau_t)}. \]  

(14)

From the labor market equation, we know that \( w_t = W_t/P_t = \mu Y_t \), and any demand is going to be satisfied at this wage. The equilibrium output is, thus, given by

\[ Y_t = \frac{\gamma - 1}{\gamma \mu (1 + \tau_t)}. \]  

(15)

The nominal price level can be obtained as

\[ P_t = \frac{M_t}{Y_t} = M_t \frac{\gamma \mu (1 + \tau_t)}{\gamma - 1}. \]  

(16)

The expected growth rates are

\[ E(g_Y) = -E(g_{1+\tau}), \quad E(\pi_t) = g_M + E(g_{1+\tau}) \]  

(17)

This shows that a permanent increase in the tax will imply a full and immediate inflation pass-through under flexible prices.

**B.3 Numerical solution algorithm**

This subsection describes our numerical solution algorithm. It consists of two parts.

---

43 We are also using the flexible price solution as starting values for the iterative procedures in our numerical solution method.
First, we solve for the steady-state aggregate variables $\pi^{SS}$, $w^{SS}$ and $\Gamma^{SS}$. As we assumed no aggregate uncertainty, aggregate variables will converge to their steady-state values. The steady-state inflation rate is equal to the growth rate of money stock: $\pi^{SS} = g_M = g_{PY}$. Then we calculate the steady-state real wages ($w^{SS}$) and the distribution of firms over their idiosyncratic state variables ($\Gamma^{SS}$) with the following iterative procedure:

1. We start with a guess for $w^{SS}$, $w_0$. Initially, this guess is equal to the flexible-price steady-state of $w$, that we can calculate analytically (see the previous subsection).

2. Given this guess and the steady-state inflation rate, we use a fine grid on relative prices\footnote{We have 2,000 gridpoints when idiosyncratic shock innovations are permanent (i.e., when the normalized relative price is the only idiosyncratic state variable).} and idiosyncratic shocks\footnote{In the temporary idiosyncratic shock case (when the idiosyncratic shock is also a state variable) we have 101 gridpoints for the quality shocks, and 500 for the relative prices.} to solve for the optimal pricing policies of individual firms. We use value function iteration with quadratic and linear interpolation.

3. With the resulting policy functions, we calculate the steady-state distribution of firms over their idiosyncratic state variables. For this, we use the same set of grids as for the value function iteration. We again do this numerically: starting from a uniform distribution, we calculate the resulting distribution after idiosyncratic shocks hit, and also after firms re-price. Then again calculate the resulting distributions after a new set of idiosyncratic shocks and new re-pricing. We do this until convergence.

4. We calculate the (Dixit-Stiglitz) average relative price in the resulting steady-state distribution. If this is smaller (larger) than 1, then we increase (decrease) our initial guess ($w_0$) of the real wages.

5. We repeat these steps until the average relative price in the calculated steady-state distribution equals 1.

In the second part of our numerical algorithm, we calculate the equilibrium paths of aggregate variables after an unexpected shock at $t = 0$ to the money supply, assuming that initially, all aggregate variables were in their steady states. We calculate the equilibrium paths of $\pi$ (inflation), $w$ (real wages) and $\Gamma$ (distribution of firms over their idiosyncratic state variables) with the following shooting algorithm:

1. We assume that aggregate variables will reach their steady-state in a finite (large) number of periods, $T$. 

\[\text{...}\]
2. We start with a guess for the equilibrium inflation path \( \{\pi_1, \ldots, \pi_T\} \). Our initial guess is the full immediate pass-through.

3. Given this guess, we calculate the resulting equilibrium path of the real wages: \( \{w_1, \ldots, w_T\} \). As \( w_t = \mu Y_t \), we do this by calculating the equilibrium real GDP path, which we know from the equilibrium inflation path (and the constant nominal growth assumption).

4. Given the inflation and real wage paths, we calculate the path of value and policy functions. We do this by backward iteration from \( T \), where the economy and the value functions are assumed to converge to a steady state.

5. Starting from period 1, and using the steady-state distribution of firms over their idiosyncratic state variables as initial distribution, we use the sequence of policy functions (together with the idiosyncratic shock processes) to calculate the resulting path of \( \Gamma \), the distribution of firms over their idiosyncratic state variables.

6. From the resulting sequence of distributions, we calculate the resulting inflation path and compare it with our initial guess. If the two are different, we update our guess to the linear combination of our previous guess and the resulting inflation path.

7. We do these iterations until the resulting inflation path is the same as our initial guess.

B.4 Source of monetary non-neutrality

We decompose the impact of the monetary policy shock on the price level to three key margins of adjustment.\(^{46}\) The intensive margin characterizes adjustment in the size of the price changes, the extensive margin characterizes the change in the fraction of adjusting firms, and the selection effect measures the variation in the composition of the adjusters. The inflation rate is a function of the price gaps (the adjustments \( x_1, x_2 \) that firms would make if menu costs disappeared for a period), their hazard function \( \Lambda(x_1, x_2) \) and distribution \( l(x_1, x_2) \) (see, for example, Costain and Nakov, 2011).\(^{47}\) The inflation rate equals

\[
\pi = \int \int (x_1 + x_2) \Lambda(x_1, x_2) l(x_1, x_2) dx_1 dx_2, \tag{18}
\]

\(^{46}\)In the simple model, the marginal aggregate shock size guaranteed that the aggregate frequency does not respond to the monetary policy shock. For reasonable shock sizes, we can not rule out the impact of this channel.

\(^{47}\)The price gaps are functions of the individual state variable \( p_{-1} \) (the last period’s quality-adjusted relative price) and aggregate states that we suppress for notational convenience.
and the impact of an aggregate shock in period $t$ can be decomposed into an intensive, extensive and selection margins, as

$$\pi_t - \pi = \underbrace{\Lambda(x_1, x_2)\Delta \pi}_{\text{intensive}} + \underbrace{\Delta \Lambda(x_1, x_2)\pi + \Delta \Lambda(x_1, x_2)\Delta \pi}_{\text{extensive}} +$$

$$\Delta \left( \int \left( x_1 + x_2 \right) \left( \Lambda(x_1, x_2) - \bar{\Lambda}(x_1, x_2) \right) l(x_1, x_2) dx_1 dx_2 \right),$$

(19)

where $\bar{l}(x_1, x_2) = \int \int l(x_1, x_2) dx_1 dx_2$ and $\bar{\pi} = \int \int (x_1 + x_2) dx_1 dx_2$, and $\Delta y$ for any variable $y$ denotes difference from the steady state $y_t - y$.

The intensive margin is the product of the average frequency and the change in the average price gap: in a Calvo-model with a fixed frequency and random selection, this would be the only component of the pass-through. The extensive margin is defined here as the aggregate effects caused by the changes in the price-change frequency. It is the sum of two products: the product of the frequency increase and the average price gap and the product of the frequency change and the average price gap. For small shocks that do not influence the aggregate frequency, this term is negligible. The third factor is the selection effect, which is the consequence of ‘new’ price changers having higher than average price gaps. The third term expresses this by measuring the increased correlation between the price gap and the adjustment hazard after a shock.

To obtain a single price-flexibility measure from impulse responses, we calculate an average pass-through of a shock. For this, we first calculate a ‘marginal’ pass-through ($\gamma^m_t$) for each period $t$, measured as the inflation effect in period $t$ relative to the fraction of cumulative money shocks yet to be passed through:

$$\gamma^m_t = \frac{\pi_t - \bar{\pi}}{(\sum_{i=0}^{t} \Delta m_i - \sum_{i=0}^{t-1} (\pi_i - \bar{\pi}))}.$$  

(20)

We weight these marginal pass-throughs based on their relative size ($\omega_t = (\pi_t - \bar{\pi})/(\Delta m/(1 - \rho))$) to arrive at a measure of aggregate price flexibility:

$$\bar{\gamma} = \sum_{t=1}^{T} \omega_t \gamma^m_t.$$  

(21)

This weighted average marginal pass-through is one in case of full price flexibility. In the Calvo model, the measure equals to the periodic marginal pass-through, which is constant over time.

The impulse responses presented on Figure 7 generate price flexibility ($\bar{\gamma}$) of 40 percent in our baseline mixed normal model, and 12, 62 and 11 percents in the Poisson, Gaussian, and Calvo models, respectively. We find that the difference in the selection effect drives the difference across
models. For small money growth shocks, extensive margin effect is negligible. The intensive margin effect, in turn, is very close in each model, the differences coming only from the variation in the frequency of price changes at 0 inflation rates (they are 15.5 percent in our baseline and 17.5 percent in the Gaussian models, but only 11 percent with Poisson and Calvo). The selection effects, however, are very different: contributing to the measured pass-through by 24 percentage points in our baseline mixed normal case and by 44.5 percentage points in the Gaussian case, while they are only 1 percent in the Poisson case. This confirms that similarly to the simple model, the extent of stochastic volatility strongly influences the extent of monetary non-neutrality of the models through its impact on the selection effect.

B.5 Calibrated parameters of the robustness exercises

This section presents the calibrated parameters of the robustness exercises. The parameters were calibrated to match the observed frequency of price changes and size of absolute price changes (in all), the kurtosis of price changes (Poisson and mixed normal) and interquartile range of the absolute price-change distribution (mixed normal). The table shows that there are some differences between the parameters across different models, but this does not lead to qualitative differences between the predicted monetary non-neutrality as Section 5 shows.

Table 7: Calibrated parameters of the robustness exercises

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multi-product, aggregate changes</td>
<td>Multi-product, aggregate changes</td>
<td>Multi-product, aggregate changes</td>
</tr>
<tr>
<td></td>
<td>0 inflation calibration</td>
<td>positive inflation calibration</td>
<td>positive inflation calibration</td>
</tr>
<tr>
<td></td>
<td>Mixed  Poisson  Normal</td>
<td>Mixed  Poisson  Normal</td>
<td>Mixed  Poisson  Normal</td>
</tr>
<tr>
<td>( \phi )</td>
<td>2.4%  1.6%  5.0%</td>
<td>2.3%  1.0%  5.0%</td>
<td>2.4%  1.9%  5.0%</td>
</tr>
<tr>
<td>( \sigma_A )</td>
<td>4.3%  4.4%  3.8%</td>
<td>4.4%  4.7%  3.8%</td>
<td>4.2%  4.3%  3.8%</td>
</tr>
<tr>
<td>( p )</td>
<td>91.2% 90.6% 0</td>
<td>92.3% 90.8% 0</td>
<td>91.3% 90.7% 0</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>8.8%  0  1</td>
<td>9.7%  0  1</td>
<td>7.5%  0  1</td>
</tr>
</tbody>
</table>

|               | Stochastic menu cost            | Stochastic menu cost           | Stochastic menu cost           |
|               | aggregate fluctuations, 0 inflation calibration | aggregate fluctuations, positive inflation calibration | aggregate fluctuations, positive inflation calibration |
|               | Mixed  Poisson  Normal          | Mixed  Poisson  Normal         | Mixed  Poisson  Normal         |
| \( \phi \)    | 1.0%  0.7%  2.45%               | 0.9%  0.2%  2.43%              | 0.9%  0.7%  2.44%              |
| \( \sigma_A \)| 4.3%  4.4%  3.6%                | 4.5%  4.8%  3.6%               | 4.3%  4.3%  3.6%               |
| \( p \)        | 90.8% 90.5% 0                  | 91.8% 90.7% 0                  | 90.8% 90.5% 0                  |
| \( \lambda \)  | 7.4%  0  1                      | 8.7%  0  1                     | 6.4%  0  1                     |
| \( \kappa \)   | 0.8%  0.8%  0.8%                | 0.8%  0.8%  0.8%               | 0.8%  0.8%  0.8%               |
Table 8: Forecast equation parameters in models with aggregate fluctuations (zero inflation calibration)

<table>
<thead>
<tr>
<th></th>
<th>Multi-product</th>
<th>Stochastic menu cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mixed</td>
<td>Poisson</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.41</td>
<td>-0.41</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>0.49</td>
<td>0.88</td>
</tr>
<tr>
<td>$\alpha_{m^2}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha_{m^3}$</td>
<td>-0.2</td>
<td>-1.0</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>

$\alpha_{m^2} \cdot 100$ and $\alpha_{m^3} \cdot 10000$ are shown.
Table 9: Random menu costs: moments in the months with tax changes

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
<th>Poisson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+3%</td>
<td>74%</td>
<td>77%</td>
<td>165%</td>
<td>40%</td>
<td>81%</td>
</tr>
<tr>
<td>Inflation pass through</td>
<td>+5%</td>
<td>99%</td>
<td>97%</td>
<td>142%</td>
<td>47%</td>
<td>102%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>33%</td>
<td>27%</td>
<td>12%</td>
<td>38%</td>
<td>12.6%</td>
</tr>
<tr>
<td>Frequency</td>
<td>+3%</td>
<td>52%</td>
<td>39%</td>
<td>80%</td>
<td>18%</td>
<td>37%</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>62%</td>
<td>62%</td>
<td>89%</td>
<td>23%</td>
<td>62%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>27%</td>
<td>20%</td>
<td>11%</td>
<td>16%</td>
<td>13%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>+3%</td>
<td>10.6</td>
<td>13.0</td>
<td>24.4</td>
<td>5.8</td>
<td>11.7%</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>8.1</td>
<td>18.6</td>
<td>26.5</td>
<td>10.4</td>
<td>18.1%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>9.2</td>
<td>6.4</td>
<td>3.4</td>
<td>3.9</td>
<td>4.0%</td>
</tr>
<tr>
<td>Abs. interquartile range</td>
<td>+3%</td>
<td>5.1%</td>
<td>2.3%</td>
<td>1.9%</td>
<td>2.8%</td>
<td>2.6%</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>5.9%</td>
<td>2.9%</td>
<td>1.9%</td>
<td>3.2%</td>
<td>2.9%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>5.0%</td>
<td>4.3%</td>
<td>11.3%</td>
<td>2.8%</td>
<td>12.7%</td>
</tr>
<tr>
<td>Absolute size</td>
<td>+3%</td>
<td>6.5%</td>
<td>7.8%</td>
<td>6.7%</td>
<td>10.4%</td>
<td>8.7%</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>9.0%</td>
<td>8.2%</td>
<td>7.6%</td>
<td>10.9%</td>
<td>8.5%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>8.6%</td>
<td>8.1%</td>
<td>10.6%</td>
<td>10.5%</td>
<td>9.3%</td>
</tr>
</tbody>
</table>
C Background on the VAT shocks in Hungary

C.1 Macroeconomic environment in Hungary between 2002-2006

In this section, we describe the macroeconomic environment in Hungary around the value-added tax changes between 2004-2006.

Between 2002 and 2006, Hungarian real GDP grew by an average of 4.2 percent per year. As panel A of Figure 10 shows, this growth rate was remarkably stable: the yearly growth rates fluctuated between 3.9 percent (2003 and 2006) and 4.8 percent (2004). Meanwhile, the core inflation (see panel B) averaged at a low 4.1 percent.\(^{48}\)

This relatively quick real growth can partly be explained by the rapid growth rate of private debt: the approximately 30 percent private debt-to-GDP ratio in 2002 increased steadily to around 55 percent by 2006 (panel D).\(^{49}\) In parallel, the budget deficit was also running high (between 6.4 percent of GDP in 2005 and 8.9 percent of GDP in 2003), and government debt increased from 52 percent of GDP in 2002 to around 62 percent of GDP in 2006. Under these circumstances, it is hardly surprising that household consumption (panel E) and the volume of retail trade also increased steadily during this period.\(^{50}\)

The 2004 value-added tax increase was motivated by a minimum tax rate requirement of the European Union. It was introduced in parallel to other related measures, some tax increases, some tax decreases with minor net effect on the net personal disposable income (see panel F). The 2006 value-added tax changes were aimed at simplifying the tax code. The tax decrease was implemented in January, with only minor additional tax measures, partly to improve the popularity of the government before the April 2006 general elections (which resulted in the government being re-elected). The tax increase in September 2006 was announced jointly with other measures which were aimed at reducing the budget deficit. Most of the tax measures influencing personal income took effect only in January 2007, while the VAT-increase (September) and some regulated price increases (e.g., heating gas, electricity) were implemented mid-year.

The inflation targeting central bank communicated explicitly that it would not react directly to the value-added tax changes, as their direct effect would disappear from the inflation rate at the policy horizon. It added that it would monitor any second-round effects through changing expectations. Indeed, there are no immediate changes in the policy rate right after the value-added tax changes, and during this period it seems that the central bank was mostly responding to depreciating exchange rates (see panel C). The exchange rate was mostly stable during the period, temporary shocks to it were counteracted by the interest policy of the inflation targeting central bank.

\(^{48}\)Panel A shows the level of GDP normalized to 100 in 2001Q4. Panel B shows the month-on-month core inflation rates. Their average is 0.345 percent, or approximately 4.1 percent annually.

\(^{49}\)Panel D shows the sum of household and corporate debt as a fraction of GDP.

\(^{50}\)Panel E shows the level of households’ consumption expenditures normalized to 100 in 2001Q4.
One could claim that factors missing from our model had a significant influence on the observed pass-through of the VAT-changes. First, we saw an approximately 10% depreciation of the local currency, the Hungarian Forint (relative to the Euro) in the summer of 2006, i.e., just before the five-percentage-point tax hike. However, this depreciation was only temporary and counteracted by interest rate hikes by the exchange-rate smoothing central bank. Furthermore, given the long time lag at which exchange rate movements pass through into processed food prices in Hungary, we can safely assume that the exchange rate had a minor impact on the CPI developments. Second, there were a series of regulated electricity and gas price increases in August and September of 2006 (together with the VAT-increase). However, they affected only the consumer prices of gas and electricity and had no impact on producer prices, so this could only have had a minor impact on retail firms pricing behavior in September 2006. Finally, fiscal policy measures in parallel with the VAT-changes have not had a significant impact on the net disposable income (see panel F), so their effect on prices could only have been minimal. All in all, we argue that most of the observed movements in the inflation rates were due to the VAT-changes, and other factors can be safely disregarded.

C.2 A 5 percentage-point tax increase in 2009

Although the product groups affected by the 2004 and 2006 VAT changes were similar, these groups were not identical. This may lead to composition bias in estimating moments. In this subsection, we use evidence from a five-percentage-point VAT increase in 2009 to evaluate the possible size of this bias in estimating the asymmetry in inflation effects in 2006. This additional evidence helps us because now we have products that were hit by both a VAT-decrease and a VAT-increase, so we can directly compare their price responses. The difficulty, however, is that during the 2009 increase, the economy was undergoing a severe recession that might have a substantial impact on the inflation pass-through. Controlling for the business cycle effects implies a comparable level and asymmetry of the pass-through as in our baseline experiment.

In July 2009, in an attempt to increase government revenues during the financial crisis, Hungarian authorities decided to increase the by now unified VAT-rate of 20 percent to 25 percent. As the second column of Table 10 indicates, 102 of the 128 products in our original processed food sample were hit by this new five-percentage-point tax increase.$^{51}$ Our estimate for the inflation pass-through of this tax change is 56.6%, which is relatively small. One possible explanation for this moderate pass-through is the ongoing massive recession (6.8 percent fall in Hungarian real GDP

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$^{51}$The remaining 26 items (all of the basic food items) got into a newly created 18 percent VAT-category. In essence, this created a multiple-rate VAT-system again with rates 18 percent and 25 percent, but now a much smaller proportion of the consumption basket had the lower VAT-rate than before 2006.
in 2009; as opposed to the 3.9 percent real GDP growth in 2006).

Table 10: Inflation pass-through in 2009 and 2006

<table>
<thead>
<tr>
<th>Tax inc in 2009</th>
<th>Inc09-Dec06</th>
<th>Inc09-Inc06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of items</td>
<td>102</td>
<td>29</td>
</tr>
<tr>
<td>CPI-weight</td>
<td>10.75</td>
<td>3.37</td>
</tr>
<tr>
<td>Infl PT in 2009 (+5%)</td>
<td>56.6%</td>
<td>68.8%</td>
</tr>
<tr>
<td>Infl PT in 2006 Jan (-5%)</td>
<td>–</td>
<td>32.6%</td>
</tr>
<tr>
<td>Infl PT in 2006 Sep (+5%)</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

The third column of Table 10 contains information about the inflation pass-through for those 29 processed food items that were hit by both the 5 percentage-point tax decrease in January 2006 and the 5 percentage-point tax decrease in July 2009. Even in this group (not subject to composition bias), we see substantial asymmetry, 32.6 percent vs. 68.8 percent. This happened even though the pass-through in 2009 was in general much smaller than in 2006: according to column 4, for the 73 items that were hit by both tax increases of September 2006 and July 2009, the respective pass-throughs were 51.1 percent and 88.0 percent. So the substantial asymmetry in column 3 is likely to be underestimated due to business cycle effects, thus our original estimate (33 percent vs. 99 percent) seems reasonable.\footnote{If we assumed that the 2009 pass-through was proportionally lower for each single product (i.e., only 51.1/88.0=58.1\% of the 2006 pass-through), then the asymmetry in the third column would be 32.6 percent vs. 118.5 percent, not far from our original estimate.}

D Asynchronous variation in price dispersion

In this section, we offer some suggestive evidence supporting our choice of an idiosyncratic shock that is drawn from a distribution with random volatility. In particular, we show that the standard deviation of price changes varies over time, and it does so asynchronously across products. This is inconsistent with standard menu cost models, in which a price change induced by idiosyncratic shocks has uniform dispersion. We show evidence for this using three micro-level price data sets. The first is our baseline dataset underpinning the calculation of the Hungarian consumer price index (CPI). The data is available between January 2002 and December 2014 and covers over 700 narrow product categories from non-regulated products of the CPI (covering over 70 percent of the aggregate CPI basket). We calculate the weighted\footnote{We use the consumption share of each product groups in the consumption expenditure survey as weights.} standard deviation of log price changes ($dp_{it} = \log p_{it} - \log p_{i,t-1}$) in 107 3-digit product groups (e.g., milk).\footnote{We are using 3-digit product categories instead of 5-digit categories because it allows us to raise the number of observations within each category to minimize the small-sample bias. On average, we have 68 prices per category} We detrend each product-
level time series and clean them from seasonal variation and the contemporaneous impacts of VAT changes. To measure the level of asynchronicity across products, we regress each product-level standard deviation separately on the aggregate standard deviation, and calculate the fraction of the time-series variation in dispersion that is explained by the aggregate, on average. We find that the weighted standard deviation of the whole cross-section explains only 2.5 percent of the product-level volatility, on average. This observation suggests a high level of asynchronicity among the dispersion of different products.

The relatively low number of observations per product groups can raise the potential concern of a small-sample bias. To address these concerns, we confirm our findings using a second data set of detailed barcode supermarket prices with thousands of observations per category. The data is from the U.S. and is available over 2001-2011. The marketing company IRI collects it, and includes weekly observations of average prices for 31 food and healthcare product categories (for example, carbonated beverages) sold in significant supermarkets all across the US (Bronnenberg et al., 2008). The data set is unusually large, it includes over 200 million observations yearly of more than 150 thousand different products in 7900 stores.

We restrict attention to three major markets: New York, Chicago and Los Angeles. As before, we calculate weighted standard deviations of price changes across product categories and the whole cross section using yearly revenue shares as weights. The size of the dataset ensures that we have more than a thousand price change observations for each category each month, so the small-sample bias is minimal. We seasonally adjust and detrend the data.

Figure 11 shows the weighted standard deviations of the price changes across the whole cross-section and three product categories of beer, carbonated beverages, and coffee. The figure shows that the dispersion is variable and is asynchronous across product categories. The cross-sectional dispersion is high, and the time series is characterized by weekly cycles.

---

55 We seasonally adjust the data using monthly dummies, and filter out the immediate impact of value-added tax changes by subtracting the coefficients of time-dummies that take the value 1 in the months with the tax changes. We obtain the cyclical component of the series by subtracting a Hodrick-Prescott trend from the series with a smoothing parameter of 129400.

56 We would like to thank IRI for making the data available. All estimates and analysis in this paper, based on data provided by IRI, are by the authors and not by IRI.

57 We make some straightforward adjustments to the data. First, we round prices toward the nearest penny, as fractional prices reflect the impact of promotional sales during the week, not actual price changes. Second, we winsorize price changes at ± 1 log points to minimize the impact of outliers. Third, we impute prices forward unchanged over a spell of missing observations, if the spells are no longer than one month and the prices before and after the spell are the same. Fourth, we only consider products that are available in the whole calendar year of the observation. Fifth, we sales-filter the data by eliminating temporary V-shaped drops in prices that are reversed entirely within five weeks (Nakamura and Steinsson, 2008). To calculate monthly price-change dispersions, we pool weekly price changes within each month. Furthermore, we exclude two major product categories: photo and razors, for which we do not have data during the whole sample period. The photo category, for example, mainly included accessories to film development, which became obsolete with the distribution of the digital technology.

58 We use monthly dummies for seasonal adjustment and obtain the cyclical component of the series by subtracting a Hodrick-Prescott trend from the series with a smoothing parameter of 129400.
standard deviation explains 8.7 percent of the 29 category level standard deviations, on average. This number is higher than the value we have obtained using the Hungarian CPI, but its low value is consistent with our observation that product-level price-change volatility is predominantly driven by idiosyncratic factors.

Finally, we show that aggregate price-change dispersion in U.S. CPI microdata also explains less than 10 percent of the **sectoral** price-change dispersions utilizing moments in Vavra (2014). To underpin the calculation of the monthly US consumer price index, the Bureau of Labor Statistics collects price quotes for thousands of goods and services across US retail establishments. Each product is assigned a fixed weight, which reflects its consumption share in the consumption expenditure survey. Using this data\(^{59}\), Vavra (2014) shows that the weighted cross-sectional standard deviation of log price changes \((dp_{it} = \log p_{it} - \log p_{it-1})\) varies countercyclically over his sample between January 1988 and January 2012 for the whole cross-section of price changes and also for most sectors. Figure 12 presents the cyclical development\(^{60}\) of this moment for the whole cross-section and three randomly chosen sectors of processed food, apparel and travel\(^{61}\). As emphasized by Vavra (2014), the figure shows elevated aggregate standard deviations during each recession. The cyclical components of the standard deviations of the individual sectors are also positive, especially during the 2001 and 2007-2009 recessions. The figure also shows, however, that the cyclical behavior of the sectoral standard deviations is mostly asynchronous outside recessions. To measure the level of asynchronicity, we regress each sectoral standard deviation separately on the aggregate standard deviation and calculate the fraction of the sectoral time-series variation in dispersion that is explained by the aggregate, on average. We find that this average \(R^2\) measure equals 9.7 percent in this sample. This means that aggregate volatility shocks explain only a small fraction of sectoral volatility.

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\(^{59}\)The data is using monthly price observations from New York, Chicago, Los Angeles; excludes price changes larger than 500 percent and smaller than 0.1 percent, sales-filtered and seasonally adjusted. Please see Vavra (2014) for details.

\(^{60}\)The logarithm of standard deviation is filtered by using the Baxter-King (18,96,33) band-pass filter.

\(^{61}\)We thank Joseph Vavra for sharing his data.
Figure 10: The macroeconomic environment in Hungary

The figure plots development of key indicators in Hungary around the value-added tax changes. The 12% rate was increased to 15% in January 2004 (first horizontal bar), the 15% rate was increased to 20% in September 2006 (third horizontal bar) and the 25% rate was decreased to 20% in January 2006 (second horizontal bar). The indicators show a steady growth in GDP, retail consumption with small inflation rates (4%) and relatively stable exchange rates.
Figure 11: Cyclical variation in dispersion across goods

The figure shows the asynchronicity of the cyclical development of the product-level standard deviations of price changes between 2001-2011 among three example product categories and the whole cross section of food and health care products.

[source] IRI Academic Dataset
Figure 12: Cyclical variation in dispersion across US sectors

[source] Vavra (2014)  The figure shows the sectoral asynchronicity of the cyclical development of the weighted standard deviations of US CPI price changes between 1990-2009 among three example sectors and the whole consumption basket.